Mathematics and dialectics , V5 14-4-2020
By Joost Kircz

This essay, or better notes, is part of a lager project to try and understand the relationship between Marxism and Science and to a lesser degree technology.

Elements are:
- Sociology and history of science as expressions of how science is shaped by its social-historical environment. In a way this is the easiest part, as lots of works already exists on this issue. It is the historical materialist aspect.
- Examples from mathematics, physics, chemistry and biology. To what extent are they exemplary ingredients for Marxist dialectical thinking.
- The more philosophical aspects of method and models. How do humans as part of nature self-reflect (I think Lenin meant this in his reflection theory) and develop knowledge (see also {Kircz, 2015}. Materialism means there is life and a world also when we are not here, but the crux is how material subjects (Humans!) become able to think about, phantasize on, invent teleologically, future societies (e.g. socialism). How do humans emancipate from a repressive culture, whilst keeping all genuine human characteristics?

Below is nor a draft paper, nor a draft book: more like Engel’s DoN: notes.

Mohr and the General on the fundamentals of maths

An unfished and perhaps unfinishable working paper by Joost Kircz

Note to the text. This discussion is about the foundations of mathematics and not about applied -engineering- mathematics nor statistics.

I consider this text as trip to a “secret” goal, like unknown chambers in the Cheops-pyramid. The goals is to what extent is a human craft such as mathematics grounded in material reality and nevertheless a result of free human thinking. To that order, I read a lot of travel guides and comments on and between these guides. Somewhere a next step in understanding must be formulated. Join me in my journey; I apologize to the sometimes irritating amount of sand between your feet and teeth.

1.

The central theme (and for many “Marxists” even a tenet) is that human knowledge is a social construction and a result of historical grown situations, as we will discuss in chpt Y (on Histomat: Hessen, Grossman, Bernal, Pannekoek, etc. in gestation). On the one hand this sounds obvious, as humans are not born with ready-made theories. Humans are born with innate capacities to create theories.[1] Though, humans reflect on their being and historically develop models and theories.[2]

The discussions on language and language acquisition also play a role in understanding mathematics, a formal language in which we try to express abstract thinking about notions and mutual relationships of abstract entities. Just like the capacity to express ourselves in natural language by speech and writing, we have a capacity to develop and adapt to mathematical language(s), which often are languages of signs.[3]

Mathematics in one form or another is a language that enables us to describe -to a certain, yet unknown - extent regularities we witness around us, such as day-and-night, metabolic processes, and whatever else is defined as human knowledge, including natural language. The essence of modern mathematics is that we define mathematical entities axiomatically, hence the naming problem is pushed aside, everybody ‘agrees’ about the name of the object and its characteristics. The name is a sign. We declare notions as starting points of an investigation and if needed -due to inconsistencies, or wanted, due to new ideas- we declare something different. An important step is to endow the name with a sign which represent the object, the sign can be everything like 5 for the notion of five (apples), but also the Hebrew letter Aleph (א), the name of the infinite totality (cardinality) of all natural numbers. Having done so, we introduce operations (activities) between these objects such as the elementary arithmetical operations: addition, subtraction, multiplication, and division. We still are free to consider symmetries or not, e.g., that the result of object A times object B is the same as the result of object B times object A, which is already not the case for the object “rotation in 3 dimensions”, where the order of turning around an axis makes a difference. The reader is invited to experience this with a book.

Building further, we introduce a whole catalogue of novel notions and features of operations. The big question, many philosophers (e.g., Leibnitz) struggled with is: is there ONE overall complete (mathematical) language, which encompassed all human understanding? In other words are we able to represent all we know and will know in the future in a single formal model. Or, more relaxed, is mathematics a toolbox with various approaches, though - in principle - each tool always follows strict demands of internal consistency and rigour? These demands make it a non-natural language such as our human

[1] Comparable with, for instance, the discussion of the possibility of an innate language faculty, suggested by Noam Chomsky   (1928-), given the fact that young children learn their native language with such a remarkable speed, independently of the complexity of those languages. Because naming things is not really the essence of inter- human communication, Chomsky directed his investigations to grammar, the structural aspect of communication and suggested a co-called generative grammar and emphasised syntactic structures.
[Quotes, e.g. from rules & representations/// the tension between structure and naming].

[2] See e.g. the masterly recent book of {Jürgen Renn, 2020} .

[3] [Here ref to Anna Sierpinska??]

speech which is far from consistent, making human communications so interesting and resourceful.
As a given, we humans have the biological capacity to create mathematics by abstract thinking, a human quality waiting for explanation. Does nature "allow" humans to drum-up mathematics or does nature "endow" humans with a glimpse of some intrinsic natural operations? The last enigma is linked to the notion of natural laws, waiting for us to be discovered, provided that we fully understand the objects and the interactions of the entities residing in that law.
Historically, mathematics developed out of human activity, including simple, conversation, sharing knowledge, and, importantly, exchanging goods. The last role induced the fields of book keeping and accountancy, which on its turn induced whole sub-fields of mathematics. There is strong evidence that it was trade which induces writings systems, as is convincingly put forward by {Schmandt- Besserat, 1999} in the case of the Mesopotamian script that emerged from tokens (small clay counters).
In the framework of historical materialism, it is human activity, and in particular productive activity and the exchange of goods that stimulated the first phase of mathematics. A second source next to trade is agriculture and husbandry where seasons and recurring weather patterns of which humans are dependent. We envision a mutually influencing of human praxis and abstract mathematical theory and conceptualization, as well as the reverse, the attempts to applying mathematical knowledge in understanding social or scientific processes. The to and fro between experiences and abstracting, between regularities and mathematical rules are fully intertwined and it is in principle undoable to even consider a one-to-one or even a one-to-many (nor many to one) projection from one side of the (not necessarily flat) coin to the other. Even, provided that both sides constitute one and the same coin, and not different currency. We deal constantly with partial projections (or "mappings") between human experiences and formal models, and hope that this patchwork quilt will ultimately lead to an overarching understanding of humanity as part of nature. In other words in every context the mappings can be different and not necessarily overlapping. Look for instance at the various geographical mappings such as the various cartographic systems (graphically representing a geographical area).[4] So, models of projection type one, tell us something different then models of projection type two. The traditional Kantian idea is that all these "pictures" together approach the unknowable object as such. But does the object itself "knows" this and allows us to approach it asymptotically? How stable is the unknowable object? To me, the object seems to be a morphing of some real phenomena with the changing model we humans make. In that case we don't asymptotically approach the true/real object, but continuously the real creates, imposes, or induces a novel mental picture in our brain.

We are facing problems on more than two sides of the proverbial coin. On the one hand we constantly experience, that is to say, discover novel phenomena, even if we, in an earlier period declared them non-existent, such as biological magnetism. On the other hand, based

---

[4] A great example is the catalogue of the various representations of public underground systems. Showing that a clear overview for traveling from A to B, doesn't mean a clear representation of the actual distance between A and B {Ovenden, 2005}.

on acquired knowledge, theories and experiences, we are able to invent “extra-natural” contraptions such as a bike, a thing that would never come around by natural evolution, even if a rolling stone or tree trunk might be the genesis of the notion ‘wheel’. We are able to invent theories that, remarkably effectively, help us in surviving and understanding. In this category we have non-formal theories such as religion, pure formal theories such as relativity theory, and even models which wait for transcending a first phase of theory formation, such as evidence-based medicine with its present natural endpoint in statistics. In our present-day culture, we are under heavy pressure to try and find formal theoretical modelling everywhere. In particular in sociology and psychology, there is a strong pressure to formalise and make it a “science”, where science stands for the methods that make physics so successful. These and other fields desperately try to mimic natural sciences, but unfortunately hardly exceed statistical methods and other applied approximations.[5]
The last sentence is certainly not a jab to the social sciences and the humanities. The opposite is true, the real question is to what extent it makes sense to apply the successful methods of the natural sciences, which after all are already heavily curtailed mathematical theories, in those fields. And even stronger, is the formal logical approach of modern mathematics the right starting point to invent theories in those fields.

[5] Some example is needed here. Certainly a pinnacle of human thinking is the highly successful General Theory of Relativity. Besides that this ‘general ‘ is a misnomer, it is build up from more than a fist full of necessary axioms, and that every step in its development limits possible choices, in order to meet its goal. In that sense the theory is certainly rigorous, axiomatic and consistent, but it uses and even shaped mathematical models, in particular tensor and manifold theory, as ingredients and in that sense is certainly not an independent fundamental mathematical theory. And, even stranger, it caters for the GPS in our car

Why should people try to mimic, e.g. physics? The present director of the Princeton Institute for Advanced Science and former president of the Royal Dutch Academy of Science (KNAW), the theoretical physicist Robbert Dijkgraaf, is a talented populariser of science who writes regular columns in the Dutch daily newspaper NRC- Handelsblad. Disciplines outside the exact sciences that do not deal with mathematics every day are somewhere between these two emotional opposites of formula fear and formula malice. That *'jalousie de métier'* sometimes manifests itself in the suppressed desire to summarize one's own discipline into one rule. But if sciences such as business administration or psychology evaporate on a mathematical fire, does then not escape all valuable components in the rising steam clouds// (1)
He goes on in explaining that in a formula the is equal "=" sign connects two different notions. The "=" sign is not an object so Dijkgraaf agues but an activity: the verb be [a is b]. And he concludes with: The equal sign is not a 'being-right' sign (2). {Dijkgraaf, 2008a}. In a later column, he talks about ''number fetishism *(cijferfetisjisme)* and criticises the relative simplicity of physics models in comparison with fewer straight forward issues such as quantum computing and big data*:* Not the bits have a central position, but the way information and algorithms are being used and shared (3). {Dijkgraaf, 2018b}. Note that this say nothing about the use of numbers as tools for counting.

(1) *"Vakgebieden buiten de exacte wetenschappen, die niet dagelijks met wiskunde omgaan, bevinden zich ergens tussen deze twee emotionele tegenpolen van formulevrees en formulenijd. Die ' jalousie de métier' uit zich soms in het onderdrukte verlangen de eigen discipline ook in één regel samen te vatten. Maar als wetenschappen als bedrijfskunde of psychologie op een mathematisch vuurtje indampen, ontsnappen alle waardevolle onderdelen dan niet in de opstijgende stoomwolken".*
(2) "*Het isgelijkteken is geen hebgelijk teken"*
(3) *"Niet de bits staan centraal, maar de wijze waarop informatie en algoritmes worden gebruikt en gedeeld.*

This embracing of mathematics and formal, algorithmic, calculations, certainly has to do with the present hegemonic ideology that every human activity has to be quantified as if it operates as a commodity on the world market, which is the central tenet of the capitalist mode of production and hence culture. Paraphrasing Hamlet: "to have or not to have is the question". But this question is already a limp metaphor, as it suggests a formal logical opposition between yes and no, whilst in many cases such an opposition is nonsense. However, in money questions, this formal logical opposition certainly reflects lots of misery.

One of the great problems of our time is that by the explosion of science and technology over the last two centuries, a comprehensive overview of these fields is almost impossible. Was it possible for voracious readers like Karl Marx (1818-1883) and Friedrich Engels (1820-1895), to stay abreast of the development in contemporary science, biology, and technology by reading original works? The answer is: hardly, and the time of the universal continental intellectuals, such as Alexander von Humbolt (1769-1859) is over. In our time, if we have the same aspiration, we have to rely most of the time to introductory texts and popular science books. Nothing wrong with the last category, but many of them are not more than an enthusiastic treatise about a difficult subject and many authors feel the need to phrase it in exuberant terms, as if we deal with acrobats in a circus, and/or vacuum cleaner's salesmen.[6]

Obviously, we cannot do without simplified or overstretched examples to explicate difficult issues, but unfortunately after we reach breathlessly the final chapter of many a popular science book, we often still don't have an understanding of why it is important? Except that apart from (perceived) applications, or that famous scientists find it important, and the claim that this particular subject has great importance for human life, augmenting fashionable sales claims, such as that it is important for instance to understand the reasons for the origins of the universe, or its potentials for an all problems solving quantum computer. Part of this avalanche of popular science books, often strongly overlapping in content, results from the way academic research is funded. Not only must present day academic research claim "valorisation" that is to say immediately implementable on the commodity markets or it must have perceived intrinsic humanistic values such as the - semi-religious - notion of a theory of everything and the final answer to all questions {Mirowsky, 2011}.[7] Happily, there is a segment of brilliant introductory books to all sub-fields in maths and the natural sciences (including biology) and were needed, I will refer to them.

The success of all the new sciences in 19th c such as: thermodynamics, electromagnetism, organic chemistry, geology, and also Darwin's precursor to modern genetics, filled the world with optimism and the idea that if "correctly" (what does that mean?) applied, humanity would overcome war and misery. This optimism was a strong impetus for the believe that socialism must be based on solid theory and then socialism would be able to overcome, in an organized way, the fight of everybody against everybody. Not only tried M&E to create a scientific socialism against all kinds of idealist -good willing- dreamers, it also became one of the firm believes that the organisation of society must be based on a conscious plan and that such a plan could be hammered out by proper use of science and technology.[8]

In that perspective the role of mathematics as abstract language is important, as it suggests an all-encompassing way of dealing with a great variety of issues. Paul Lafargue (1842-1911) {Lafargue, 1890} in *Reminiscences of Marx*, says: "Besides the poets and novelists,

[6] In physics an excellent example is {Wilczek, 2015}, a very bad example is the much cited {Hawking, 1988}. Actually, I got the last book as a present from the famous physicist Emil Wolf, visiting him in Rochester NY, to read during my flight to California: "phone me immediately if you understand it". I didn't phone him.

[7] The informed reader will smell my disbelieving in string theory, despite the beauty of its mathematics and the many popular accounts. For a well written critiques on String Theory see {Smolin, 2007} and {Woit, 2006}, and for a famous defence {Greene, 2000, 2004}.

[8] Interestingly, in the anthology *K. Marx, F. Engels, V. I. Lenin, On scientific Communism*, not one reference to any science is mentioned in the 500 plus pages. {Marx ,1967}

Marx had another remarkable way of relaxing intellectually – mathematics, for which he had a special liking. Algebra even brought him moral consolation and he took refuge in the most distressing moments of his eventful life. During his wife's last illness he was unable to devote himself to his usual scientific work and the only way in which he could shake off the oppression caused by her sufferings was to plunge into mathematics. During that time of moral suffering he wrote a work on infinitesimal calculus which, according to the opinion of experts, is of great scientific value and will be published in his collected works. He saw in higher mathematics the most logical and at the same time the simplest form of dialectical movement. He held the view that science is not really developed until it has learned to make use of mathematics."
One can interpret this as that he only considers a science as science, if it can be put in formal rigorous theory. The next question is then to what extent modern analysis (as was then known as higher mathematics) is the correct -and even dialectical- theory?[9]

2.
Within this mood, Engels attacked Eugen Karl Dühring (1833-1921), an important ideologue with a strong influence in the young German social-democratic movement. The direct motivation for raising alarm was the discussion around the party conference of the German social-democrats in 1875 in the city of Gotha (22-27 May, 1875).[10]
On this congress a compromise programme known under the name *Gothaer Programm* was voted. This programme was sharply criticised by Marx and Engels in their famous *Critique of the Gotha Programme* written in the same year and intended for the leadership of the so-called *Eisenachers*, named after the city in which SAPD was founded. {K. Marx. (1875, as from p. 75 'Critique of the Gotha Programme'. Engels published the text in 1891 in *Die Neue Zeit*, vol. 1, 18}
Marx and Engels critique on the opportunistic aspects in the Gotha Programme became one of the corner stones for the development of "Marxist" politics.

Dühring was a remarkable Berlin ideologue and self-declared genius, who published within a very short time span a series of books ranging from titles such as *Kapital und Arbeit* (1865), *Naturliche Dialektik* (1865) , *Kritische Geschichte der allgemeinen Principien der Mechanik* (1972), *Logik und Wissenschaftstheorie* (1876) and a bit later his pre-fascist book *Die Judenfrage als Racen-, Sitten- und Culturfrage mit einer weltgeschichtlichen Antwort* (1881). {See also: Karl Muller, 2009}.
Against opportunism and romantic pipe-dreams in the social-democratic organisations, Engels set out to define "Scientific Socialism" as a way to systematically develop socialist theory. Dühring's works became a pretext to systematise socialist thinking; " On the one

[9] Happily M&E, who were music lovers never, became aficionados of J.S. Bach's formalistic highly structured music, the way many mathematicians are. {Lindley 2010}.
[10] On this congress two parties: the "Sozialdemokratischen Arbeiterpartei (SDAP)", under the leadership of o.a. August Bebel (1840-1913) and Wilhelm Liebknecht (1826-1900), merged with the larger "Allgemeinen Deutschen Arbeiterverein (ADAV)" of Ferdinand Lasalle (1825-1864) into the "Sozialistischen Arbeiterpartei Deutschlands (SAPD)".

hand it gave me, in connection with the very diverse subjects to be touched on here, the opportunity of setting forth in a positive form my views on controversial issues which are today of quite general scientific or practical interest." {Engels, 1878, p. 6.}. Engels, with the assistance of Marx wrote a monumental *tour d' horizon* in a very short period of time: *Herrn Eugen Dührings Umwälzung der Wissenschaft* [11]. Engels he tried to address a great variety of subjects from economy, philosophy, history, and natural sciences into a comprehensive whole. This polemic work became after its 3rd edition of 1894, a foundational text in Marxism and the central educational text in historical materialism and dialectics for generations of socialists. Obviously, it is quite easy to attack this book, with the new experiences and theoretical knowledge since. The real drama is not the many hand waving examples or insufficiently well researched technical subjects, but the fact that such a strong polemic and historical contextual educational book became a bible for the social democratic movement and even a holy scripture in the Stalinist culture. This, in a complete contradistinction to Engels' emphasis on self-organization and self-emancipation of the working class. An enormous literature exists on exegesis, commenting, and critique on Engels's *Mr. Eugen Dühring's revolution in science*. [12]

But the very fact that the nickname of the book is *Anti-Dühring*, is already a strong warning that the book emphasises that what we as socialists detest, instead of the first instalment of a book series under the title *Pro-Socialism*. Dialectical *angehauchter* readers will certainly state that it is in the critique on what is wrong that we will negate this false (negation of correct) knowledge by transcending it into the positive negation into a novel outlook. However, Mr. Dühring is of no importance whatsoever since more than a century, except his historical role as one of the ingredients that induced the first attempt to comprehensively try and write an introductory text in historical materialism and dialectics. What we need now is a fresh approach that includes old discussions without the misconception that an historical approach means that we have to start at the dawn of human civilisation. The same as that Marx economical critique on the capitalist mode of production uses the historical pathways of production, but that does not mean that misinterpretations in for instance his description of the economy of the ancient Egyptians {Marx, 1976, p 451} is crucial for this description of the dynamics of modern capitalism. Such an approach to history would be a splendid example of trying to press a "law" on history which consequently must lead to a well-defined future. In the 19c the notion of physical law became so powerful that it needed Rosa Luxemburg's (1871-1917) revelation (instead of an obvious conclusion) that the capitalist mode of production does not have to end in socialism, but may as well end with the total

---

[11] From the preface p. VIII: In two years (1876-78), he wrote a major work that was first printed in Vorwärts, the newspaper of the Social Democratic Party of Germany, and was brought out as a separate book in 1878 under the title *Herrn Eugen Dührings Umwälzung der Wissenschaft* (Herr Eugen Dühring's Revolution in Science—known in English as Anti-Dühring), in which Engels subjected Dühring's views to devastating criticism.

[12] It always strikes me as a typical Germanophobe Anglo-Saxonism that the German title Herr, which simple means mister or Sir, is never translated. Just watch any UK film of (TV) play in which an unpleasant German citizen appears, it is always Herr X and not mister or Sir X.

destruction of nature's crucial experiment with humanity, qq, other bifurcations in human history are never mono-causal.

Having said that, *Anti-Dühring* together with Engels' private notes on the dialectics of nature (later collected in the book *Dialectics of nature;* see below), which joined *Anti-Dühring* as sacred text in the 1930s, are rich sources in attempts to suggest contemporary understanding of the socio-historical role in the development of human society as part of nature, as well as dialectics as an alternative to formal - linear - logic as a mental tool.

What FE is doing in *Anti-Dühring* is in the first place a polemic against obscurantist system buildings like Dühring. As he explains in the preface of the 1st edition "The following work is by no means the fruit of any "inner urge". On the contrary", {Engels 1878, p. 5.} which indicates the work is not structured as an independent treatise on socialism. However, in putting down Dühring, it obviously has a distinct educational edge. The figure of this critique is twofold, not only is it a store of arguments against the adversary, it is, first and for all, an exposition of how socialists have to judge politics in all its intricacies. In the first part: 'Philosophy', the basis of the theory, what is nowadays called 'Marxism', is given. Here we find the fundamentals of Engels' dialectical thinking and their explication in various fields. In part II: 'Political economy', we get a crash course on *Capital* volume 1. Not, as in Volume 1 itself as an independent new theory, but now in contradistinction to and arguing with Dühring's work. Therewith, it becomes an educational introduction to *Capital* volume 1, written in very lively style. Part III: 'On Socialism' is a first outline on how to apply the historical dynamics, intrinsic in the dialectics, to future vistas. It is this important section, which became under the title *Socialism Utopian and Scientific* an independent programmatic pamphlet and political guideline for generations of activists around the world. {1882, expanded ed. 4th 1891. Engels 1878, 281-325.}
We see on the one hand the attack in *Anti-Dühring* , and on the other hand his notes in *Dialectics of nature* which are embryonal pieces for a larger work on the way we understand nature and the methods and their creations to understand nature. For a contemporarily reader it is important to read through the polemics and try to understand the content of the reasoning. FE had a remarkable broad knowledge of the sciences and biology of his time. But we have to take into account that this knowledge was not always the latest and hottest, and sometime was even lacking behind, e.g., in the case of maths. Furthermore, just because it is polemic, arguments are often grounded in examples and not based on a consistent theory.
FE's notes, which started even before writing *Anti-Dühring* are interesting considerations but certainly not worked out thoughts. [13]

---

[13] Philosophisches bzw. dialektisches Denken wart für Engels - und hier stimmt er mit Hegel and nicht mit Schelling überein- ein Denken das auf Begriffe, Schlüssen und Beweisen beruht. Est ist in diesem Sinne wissenschaftliches Denken. Nur unter dieser Voraussetzung vermag das philosophische Denken den Natur wissenschaftlichen Erkenntnis Prozess zu forderen. Ein grosser Teil der vorliegende Ms ist der Fragen gewidmet in wie fern die objektive Dialektik der Natur in den Naturwissenschaften widergespiegelt wird, is wie fern diese eine dialektische Inhalt haben. {Engels, 1985, p.26*-32*}.

The problem is well posed. If we start with the notion that Nature is a dynamical system in which temporal structures and forces mutually interact. Then we realise that we are confronted with a temporal development, in our present earthly case, expressed in a tentative increase in complexity on many fronts[14], we reach limits of traditional thinking and models. The overtaking of limits in our thinking and modelling is expressed in the fact that we witness regular overhauls of scientific theories. In chptY (in gestatation) we deal with this aspect of the socio-historical roots of such model changes.

The original publication of what is now branded as *Dialectics of Nature* was part of a scholarly attempt under the leadership of David Riazanov (David Borisovich Goldendakh 1870-1938) to prepare a complete collected works of both Marx and Engels (an attempt that, on a more sophisticated way, is still ongoing in the MEGA2 project). Already in 1921, on instigation of Lenin, the Marx-Engels Institute became established with Riazanov as first director.
A first publication of DoN was in 1925 in Russian and German. In 1927 an improved version saw the light in the publication *Marx-Engels Archive*, volume II. Among other articles, such as A. Deborin's, 'Die Dialektik bei Fichte", and many book reviews, it included Engels' *Dialektic und Natur*. [15] Already in the first round of discussions on a possible publication of DoN it became clear that the collection of notes were to a large extent outdated and had many flaws. This is obvious, as the notes were written in the midst the great scientific upsurge of the late 19th c. Only after Albert Einstein, so requested, wrote Eduard Bernstein that: "Dagegen kann ich mir denken, dass dieses Schrift für eine Publikation insofern in Betracht käme, als sie einen interessanten Beitrag fur die Beleuchtung von Engels' geistiger Persönlichkeit bildet", was decided to publish it. It is not known if Einstein got only a part or the whole of the - difficult readable- hand written notes. This is a correct attitude, let the reader decide what is important and don't allow censorship. But unfortunately DoN was published during the heydays of the philosophic battles between the "Mechanists" and the "Dialecticians" in the USSR.
V. V. Adoratsky, the successor Riazanov, writes in his 1934 encyclical *Dialectical materialism: The theoretical foundation of Marxism-Leninism* {Adoratsky, 1934, p. 64}: "In addition, he [FE] wrote a large work on *The dialectics of Nature*, which unfortunately he never succeeded in publishing (the manuscript however was preserved and has been published...)" .
This brings us to the understanding why such a sketchy bundle of private notes became a "weapon in the class struggle".

[14] E.g. human thinking enables the creation of invented, non-natural, contraptions that mimic, and sometimes even extent natural phenomena, e.g. Artificial Intelligence (AI) research.
[15] A full history is given Engels, 1985, vol.1 .595-598.

Dialectics in the natural sciences and mathematics

Dialectics in chemistry, which comprises a big chunk of the notes, are relatively easy to grasp. The standard and well-proven approach in the sight of cognitive limits of understanding is to firstly stretch the existing vocabulary and tools.
Example: when it became clear that we can consider chemical molecules as being composed of a number of more elementary chemical atoms, this decomposition was immediately confronted with the need for a theory of the various forms of chemical binding. Combinations of individual entities, be it atoms or molecules, cannot exist without the notion of binding. In superficial language one might say that the particle and its binding forces are a composite totality as the new, bounded, particle is again a self-contained unit, or so you wish totality. Thinking that way, you might call the intertwined opposition of particles and binding forces a dialectical unit, as particles are thought of being objects with a well limited spatial extension and binding forces are considered as force fields which reach over long distances compared to the size of the particles, this whilst they only exists together. In this case, we take particles and force fields both as material objects.
It goes without saying that the theory of chemical binding turns out to be tremendous successful and found equivalents in theories about the composition of elementary particles, the constituents of atoms and their binding forces.
They all fit the notion of a world composed of particles and fields, to be later transcended to the idea that also particles can be described by (matter) fields. The opposition between particle and force is than "solved" by field theory.
However, the question is to what extent this picture is an expression of an innate dialectics of nature or only a human approximation of the supposed dialectics of nature?[16]
The same can be said about the so-called quantity-quality law in example of the homological chain of organic molecules where adding one carbon atom to the chain, changes the character of the molecule fundamentally.

Contrary to chemistry, in the discussion on mathematics we see most clearly the difficulties. Many economists {e.g. Carchedi, 2008, 2011, 2014} stress the point that dialectics is a process of development and not just of oppositions only. It is about transcending opposite motions into a new level of reality that keeps the essence in one or the other form of its underlying original components (discussion on labour, value, and price). See also below section 9.2.
But what is the measure of all this? Do we have a "gauge" theory that proofs the cognitive undertaking of understanding and forecasting?

Obviously mathematics is a candidate. But what form of mathematics? In the course of centuries many a field within mathematics emerged and a full unity of methods and approaches is certainly not yet on the horizon, even if now set theory is the basis of all mathematics in physics. In my opinion, the Leibnizian dream will never happen as mathematics is the playing field of free human thinking *par excellence*.

[16] In the attempt to cast all aspects of knowledge into a dialectical framework it is not enough to declare a method. Hegel's *Wesen*. See chptr dialectics.

In short, mathematics is the art of thinking that has only two rules: rigour and consistency. Everybody is free to define any mathematical object and any mathematical rule, as long as the building created from these well-defined starting points is internally consistent. The fantastic fact is that some mathematical approaches turn out to be excellent tools for describing physics and its applications. To join Lenin in his opinion that all the modelling of generations of thinkers that result in real life technology gives us an objective reality. Though, this reality will always be described differently if new facts arise. In my, modern lingo, I would say. The reality that we experience is an ever changing (to quote Lenin: inexhaustible) morphing of theories and experiment.

> "The "essence" of things, or "substance", is also relative; it expresses only the degree of profundity of man's knowledge of objects; and while yesterday the profundity of this knowledge did not go beyond the atom, and today does not go beyond the electron and ether, dialectical materialism insists on the temporary, relative, approximate character of all these milestones in the knowledge of nature gained by the progressing science of man. The electron is as inexhaustible as the atom, nature is infinite, but it infinitely exists. And it is this sole categorical, this sole unconditional recognition of nature's existence outside the mind and perception of man that distinguishes dialectical materialism from relativist agnosticism and idealism". {Lenin, 1908, p. 262}

The pertinent and often posed question; why mathematics is so effective?,[17] is fundamentally ahistorical and undialectical. Over the centuries, effective modelling came to the fore as a result of social collective labour. And when the model works (for the moment) it looks like a miracle. Human mental labour created the tin-opener as well as set theory and if we forget this, indeed by opening the tin the resulting sardines in tomato sauce looks like coming from heaven and declaring that it is a miracle that the tin-opener is so effective.

Note: dialectics and physics will be a separate texts, as here we have to deal with the often bizarre discussion on quantum mechanics and what quantum mechanics actually is.

3.
Why is mathematics of interest?

Archeological research, as well as ancient texts, shows that already in an early stage of humanity people tried to get a grasp of nature by trying to formalise regularities in the heavens, day and night, the seasons, etc. The central notion of "time" emerged as measure for changing and recurring phenomena. [18]

---

[17] The most popular reference here is the paper by the Hungarian-American Nobel laureate Eugene Paul Wigner (1902-1995) {Wigner, 1960}.

[18] There exists an almost endless amount of books on the nature of the evasive notion "time": Universal time, Human time, Biological time, Mathematical time, and Relativistic time, etc. Time finds its place in calendars of which great varieties exist over the centuries. A bit technical overview of calendars is given by Deshowitz and Reingold (1997). Digging into the essential notion of simultaneity, which bastions were razed by relativity theory, is done by {Jammer, 2006}

As soon as - more or less - working methods emerged, for instance an understanding of the life spans of animals and plants as function of the seasons, two fundamental questions came to the fore. Are these regularities given by one or more unhuman intellects (Gods), or are the regularities, we experience and can express in formal language, intrinsic features of nature? Or said differently: can we speak of natural laws and are we discovering these laws as result of changing and developing socio-economic structures in history (e.g. the start of agriculture), or are these laws contingent temporal mental constructs that slowly enable us to constantly "re-programme" our understanding of the world in an ever novel, more encompassing, thought systems (also called scientific revolutions)?
In other words: with a fitting number of well-defined notions such as energy and mass, we can discover laws that work in practice. But the combination of named entities and their interactions can as well be considered as a human invention. This however is not an argument for fashionable postmodernist relativism, as the bottom line is still real phenomena and serious theories are not in frivolous lucky bags of ideas, but are tied together in a whole structure of knowledge.
More challenging is the question of the idea of basic or ontological entities, which are needed to start the whole conveyor belt of investigations. In this department we have the nagging issue of *a priori* notions, notions supposedly we cannot go around, like time, space, and causality. In the philosophy of physics, the question of *a priori* notions as stepping stone is still a current issue. {Howard, 2004}
A fundamental reason why mathematics is an important subject of investigation is because it is the field of abstract notions *par excellence*. If we want to understand abstract notions and their meaning, which is also a key element of linguistics, it may be the case that the study of a formalized system of abstract notions: mathematics can help us to attack the far more complicated field of informal abstractions in human thinking. In other words are abstractions we extract from empirical experiences equally real as the underlying matter. [19]

In mathematics the idea of numbers is an example. Numbers as object became, since times immemorial, a fertile base for mysticism, foretelling and religion. For not yet disclosed reasons, humans cannot do without numbers as soon as we need pertinent notions to deal with. This is also expressed in the dichotomy of analogue and digital representations of whatever we might think of. We see (analogue) a crowd of which we can only estimate the size (qualified as: a lot), but only by counting individuals we quantify the tensions about the number of attendees, between the claims of the organisers of the demonstration and the claims of the police.[20]

[19] See the works of Evald Ilyenkov on the Ideal and the Vygotsky school and the "activity approach'

[20] In their book: *Where mathematics comes from. How the embodied mind brings mathematics into being,*, Lakoff and Núñez (2000) try to ground the notion of mathematics as being build up out of innate cognitive -embodied- notions of numbers and counting. See special note on Lakoff and Núñez later on.

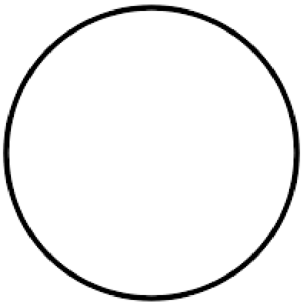
In mathematics this point is expressed by the fact that we have two separated field to compare phenomena. On the one hand we have geometry, the field dealing with shapes and forms. Geometry has its roots is the old art of measuring land by surveyors. Here we encounter the idea of continuity and how we compare various shapes. But geometry is an abstract art and presupposes already a fairly high cultural level. [21] In formalising this field, we don't need numbers, but use logical axioms as basis of our thinking. Geometry was comprehensively established in the works of Euclid of Alexandria who lived around 450 BCE. Most presumably the name stands for a person and a group of mathematicians around him who created the book series *The elements*. It is a compilation, build up from five postulates (or axioms), that covers all of classical geometry known at that time. Essentially it remains the fundamental and only approach of dealing with shapes and forms up to the 19th century. The essential importance is that this closed logical system stands out until today as an unchallenged monument of human thinking. It is no wonder that many a philosopher (such as Kant) or scientist (like Newton) took Euclidean geometry as an ontological given. Only after mathematicians in the 19th century challenged the 5th postulate which stipulates that 2 parallel lines never cross, geometry expanded to non-Euclidean geometries that allow for curved spaces. [22]

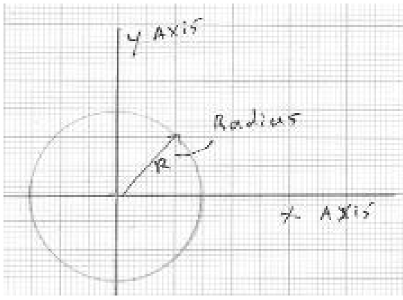
On the other hand we have the art of counting numbers: numeracy or arithmetic. This is all about the various operations between numbers, such as addition, subtraction, division and multiplication. If we expand this to operations to the manipulation of symbols we speak of an algebra.
Here, we are immediately confronted with the difficult problem of: what is a distinct number? Numbers are often metaphorical suggested as being lying on a line made out of points. In fact we use then a geometric model (a line) to depict a collection of numbers, whilst it is also possible to consider numbers as a bunch of peas in a bag, as is the start of manifold theory. Taking the line metaphor, we reach the issue of how close do numbers lay on that line and when can we speak of a continuum? (see further below). This is important as the algebraic form of a circle x^2+y^2=r^2, (if we take the coordinate points (0,0) as centre) is not the same as a circle drawn on paper, which is certainly considered as being continuous.

[21] In his famous study of the 1930s reported in *The Making of Mind*, the psychologist Alexander Romanovich Luria (1902- 1977) compared dwellers of remote Russian villages of Uzbekistan and Khirgizia. Typically, a picture of round object was identified as a plate by non-literate women, whilst subjects that experienced literacy classes identified the geometrical form of a circle. See: Michael Cole, Karl Levitin, Alexander Luria, *The autobiography of Alexander Luria, a dialogue with The Making of Mind*, Edited reprint; Psychology Press, 2010 (1979 original by Lawrence Erlbaum Associates, Inc.)

[22] Note that curved space is different from curved figures in 'normal' Euclidean - - flat - space, such as a circle drawn on a sheet of paper. Non-Euclidian space can be illustrated if we envision e.g. a triangle with one corner at the North Pole of the earth, and 2 corners at different places at the equator. The triangle is bent. The sum of the angles, so constructed, is larger than 180 degrees, as is the case in flat Euclidian geometry.

It was René Descartes (1596-1650) who in the 17th century formalised so-called analytical (coordinate) geometry, in which we introduce coordinates in geometry. As from then we can describe geometrical objects not only as independent “free floating” objects but can define them arithmetically in a coordinate system on a Euclidean space, by an algebraic formula. In case of coordinates under right angles (such as height, width and length), we speak of a Cartesian coordinate system.





Merging the fundamental different worlds into one system is still an outstanding research problem.[23]
To make life a bit more complicated in order to understand why mathematics is so fundamental in human modelling, we have to mention a few more issues, which all came to the fore in the second half of the 19century, the period that M&E tried to say something about mathematics, often following Hegel. Again, it is not the issue if M&E discuss old hat and hence their works are only of historical significance, but how we perceive problems like continuity (flow) versus discreteness (or atomism) and if we can talk about dialectical logic on the level of mathematics, yes, no, and why?

4.
On the calculus and the infinite

Readers who are socio-historically imposed, often due to their education, with the delusion that they, intrinsically, cannot understand any maths (but are hooked to their electronic devices, with all their menus) are kindly invited to continue reading. Otherwise feel free to skip this chapter as intermezzo.

[23] Since about 70 years, serious attempts are underway to try and find a geometric idea of prime numbers. After all, prime numbers, the natural numbers that cannot be written as a multiplication of other natural numbers except itself and one, might be depictable in a figure. One can develop the idea that prime numbers are a kind of coordinating the natural numbers as all non-prime numbers can be written (factorised) as a series of primes. The distribution of prime numbers is still not known, but strongly related with the idea of geometrising the natural numbers. Technically we are then speaking of p-adic numbers.

The infinite can also be an entertaining subject. Look at infinite series (just count on forever according to a simple rule). The brilliant and pedagogical website https://www.numberphile.com/ host a treasure trove of examples. E.g., the nagging quest: what is the sum of: One minus one, plus one minus one, plus one….and so on? At: https://www.youtube.com/watch?v=PCu_BNNI5x4
Or even weirder: why is the sum of all natural numbers (1+2+3+4+ to infinity) equal to -1/12 (minus 1 divided by 12), a strange result, which is actually used in e.g. string theory. https://www.youtube.com/watch?v=PCu_BNNI5x4

The calculus with matured as a technique in the 17c, and as a rigorous mathematical theory only in the 19th century, is central to work with variable quantities and intuitively with motion. Philosophical we are confronted with the notions “rest” and “motion”. How can rest pass into motion? And how can motion come to a grinding halt? If we define change, than obviously we compare two situations and we need a measure to do so. On my bike, I can compare the situation from where I start biking to a situation while biking, or stop biking. Starting point and end point are often considered as being at rest. This rest is only stasis compared to the variable I want to use for defining motion. Waiting to start a bike race is waiting for the countdown. Hence, with time (the countdown) as measure I’m not at rest, whilst with the trail as measure I’m at rest. Famous are the Zeno paradoxes that deal with this issue.
As Courant and Robbins {1996, p 399} say: “In the mathematical analysis of the seventeenth and most of the eighteenth centuries, the Greek ideal of clear and rigorous reasoning seemed to have been discarded. “Intuition” and “instinct” replaced reason in many important instances; this only encouraged an uncritical belief in de superhuman power of new formal methods. It was generally thought that a clear presentation of the results of the calculus was not only unnecessary but impossible. Had not the new science been in the hands of a small group of extremely competent men, serious errors and even debacle might have resulted. These pioneers were guided by strong instinctive feeling that kept them from going far astray. But when the French Revolution opened the way to an immense extension of higher learning, when increasingly large numbers of men [indeed mostly man -jk] wish to participate in scientific activity, the critical revision of the new analysis could no longer be postponed. This challenge was successfully met in the nineteenth century, and today the calculus can be taught without a trace of mystery and with complete rigor”.
Not only the advance of education but also the emerging industrialisation demanded clear visions and methods to pursue the many applications of the calculus. Interestingly, Marx and Engels are exponents and propagandists of the then modern wave of enlightenment. The so-called “crisis” in mathematics induced a thorough rebuilding of the field, a rebuilding that left all artisanal approaches and re-grounded mathematics on rock solid abstract and axiomatic notions. [24]
Marx understood the weaknesses of the “old” calculus pretty well as becomes clear from his notes, but the big leap forward was still in the making, while writing his comments. The

[24] The notion ‘rock solid abstractions’ is a good example of the tension between free thinking and rigorous reasoning. Aside: in General Relativity the whole notion of rock solid is discarded.

discussion in his works as well as in that of Engels is related to “solving” the difficulties by rephrasing it into a dialectics of oppositions, whilst the modern approach redefines the problem lock, stock and barrel. However, also in the formal modern approach ontological issues remain, e.g. is a continuum an existing phenomena or is it a look from afar of a sequence of immensely small entities or steps (like we see a smooth surface instead of turbulent waves, if we watch the sea from an aeroplane)? Marx seriously dealt with the issue, but based on the old fashioned textbooks he had as well as his knowledge and critique on Georg Wilhelm Friedrich Hegel (1770-1831)’s logic. See further section 4.5.

4.1 Motion

The fundamental issue is; how to describe development or progress in time (or any other variable) in formal terms. Time and motion are intertwined notions. Time is a measure for motion and motion tells us how something changes in time. With a wink to Marx we can say that Time is a universal equivalent, like Money, everything is brought back to time; which does, however, not prove that time=money. If materialists, in particular FE in his DoN, say that everything boils down to “matter in motion”, and then we immediately are confronted with the need for a proper understanding of matter as well as of motion. The notion matter is a complicated one and in normal speech it means stuff, something we can sensory feel, or e.g. sit on, or ate/drink. The term has a long history and its meaning depends on how we can make this simple term operational. In formal physics the term is closely related to inertia or acceleration: the amount of resistance to change of place, e.g. falling, in colloquial speech. Matter, in its formal form is called mass and is at the bottom of all physical theories. Matter, for that matter, as such is too fluffy to enter formal theory, but mass is a notion we can approach axiomatically such as in the Newtonian law F=mA in which the notions Force (F), (inertial) mass (m), and acceleration (A) are co-defined. {for a thorough historical discussion see: Jammer (1961) and Jammer (2000) }. Realize that matter in the meaning of stuff is philosophically distinctly different than the notion of philosophical materialism {Lenin, 1908} where we say that nature exists without human perceptions.
Motion, on the other hand is seemingly easier to formalise as the notion that some attribute of a quality changes as result of the change of some influence due to another quality. A simplest example is: I bike. “I and my bike” moves in time, means the object “I and my bike” changes place on a road: I bike from here to my meeting. The rate of change between being here and coming there is called velocity, defined as the amount of metres biked, divided by the time in seconds it took. Concretely e.g. my velocity was (on the average) 15 km per hour, means that it took me 20 minutes to reach the meeting place 5 km away. Obviously, this example is a *pars pro toto* for all types of change. In order to formalise this we use the notion of a function, an idealisation of how a varying quantity depends on another quantity. In formal language f(x)=y means that some activity, called function, working on x gives y as a result. A function, a mathematical object, tells us how a quantity of type 1 changes as a result of a quantity type 2. This means already that we allow for variable quantities opposite to discreet quantities such as: one bike, two bikes,...
In our example, my “velocity” times “the time I bike” gives “the distance I biked”. This sentence already reveals a lot. Namely, if I bike, for simplicity, with a constant velocity then the velocity is a time independent notion, while the distance covered on the road is

depending on the time I biked. Motion becomes a measure for change (between being here and coming there). It goes without saying that under realistic circumstances the motion is not constant, and there exist a rate of change. In our case: the rate of change in one dimensional space (a road) as "function" of the variable time. I start fast, stop for a pee, and go on further slowly. Although in abstract terms, nor the road axis, nor the time axis demands a measure or unit of measurement along the motion. In practical life we deal with a so-called metric. We measure space in metres and time in seconds, and all metres and all seconds are of the same size, and -most importantly- considered independent of each other. Such an approach is catered for in what we call Euclidean space, the type of mathematical space which is our standard understanding of space and time for over more than 2 millennia. The mathematical field of functions is called analysis and the field of space (and time) is called geometry. [25]

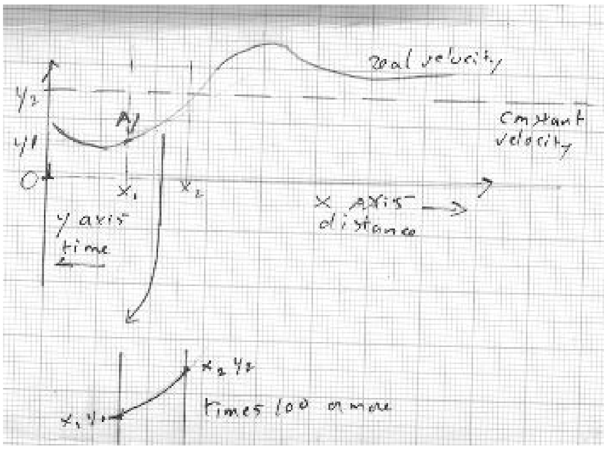
However, remain aware of the fact that formal mathematical textbooks, even if they deal with applications of mathematical techniques in other fields such as physics, marketing, or population demography, do normally only explain the how and sometimes the why of a method or technique, and most often don't entre the discussion on the limits of these techniques. Methods remain abstracted human models for describing phenomena. Most books emphasize the intrinsic development of a method as a creative process, with only scarcely attention to influences external to the to-be-solved problem at issue.
The big thing is; to what extent does a mathematical model mimics a phenomenon, or otherwise that the model is an intrinsic -ontological- characteristic of what we experience. In the case of the opinion that mathematical objects, say a triangle, is a real fundamental object (although there exist no perfect triangle in nature) we name it Platonism: the notion that such ideal constructions have a fundamental truth value, we cannot go around or beyond them. In mathematics as a field of free thinking, we can define infinite types of mathematical objects (see also below), which -if we wish so- define eternal values (for what it is worth). Here we see the difference between Hegel and Marx& Engels. Hegel took free will as starting point for his teaching of logic, in which he starts with abstract categories in order to stratify language that enables us to think in a systematic dialectics.
In the other extreme, an empiricist approach, we can say that nature as "matter in motion" or "Motion is the mode of existence of matter" {Engels 1878, p.55.} explicates behaviour and that we understand this behaviour in a systematic analysis and translate our experiences into a theory. Obviously the debate on the nature and philosophy of mathematics, despite the endeavours of the last 2000+ years, is still in its infancy. We don't (yet?) know how our brain works and how experiences and thoughts intertwine on a fundamental level.[26]

Since René Descartes (1596-1650), the field of functions and in particular the merging of geometry and algebra (the field of rules and manipulation of mathematical symbols,

[25] There are many elementary textbooks on both fields. {Alcock, 2014} is a particularly well written introduction to analysis. A more demanding excellent textbook is the evergreen {Courant-Robbins. 1996}.

[26] Though cognitive scientists claim grounding in neurology, see {Lakoff and Núñez, 2000}.

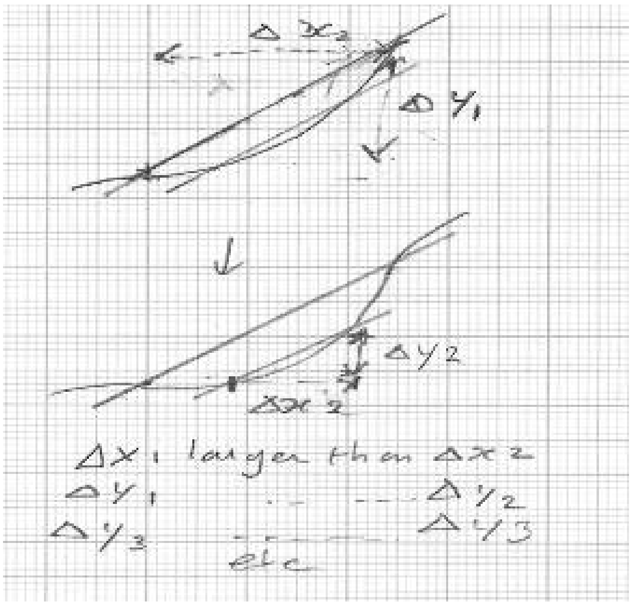
transcending simple arithmetic) exploded. It is also only since then that the now fully internalised method of picturing mathematical notions took off, immensely stimulating reasoning. Engels goes even so far as declaring: “The turning point in mathematics was Descartes' variable magnitude. With that came motion and hence dialectics in mathematics, and at once, too, of necessity the differential and integral Calculus, which moreover immediately begins, and which on the whole was completed by Newton and Leibniz, not discovered by them.” {Engels 873-1883, p. 537}. This is a sloppy comment as if a mathematical technique (which would be completely redefined and made logically consistence in Engels’ own life time) is discovered (like a new metal ore) instead of being an human endeavour: an invention. Secondly it suggest [check Hegel] that dialectics can be defined by the fact that a quality (say speed) can have two faces: a fix value and a changing value. This obviously is a reflection of the age-old question if we can talk about a velocity at a very precise moment of time.
Or even more pertinent: “Philosophy takes its revenge posthumously on natural science for the latter having deserted it; and yet the scientists could have seen even from the successes in natural science achieved by philosophy that the latter possessed something that was superior to them even in their own special sphere (Leibniz—the founder of the mathematics of the infinite, in contrast to whom the inductive ass Newton appears as a plagiarist and corrupter”. {Engels 1873-1883, p. 486. In a note: The Russian editor of DoN suggest Engels is following Hegel here}.[27]
In our example of motion, we can depict my biking in as picture, a graph, with two axes. Standard we have the distance as x-axis horizontally and time as y-axis on right angles with the x-axis, creating a so-called coordinate system. Again there is not any reason to stick to this right angle coordinate system, with the honourable name of Cartesian (after Descartes) coordinate system. In the picture we picture my biking.



Point A is at distance x1 and time y1

[27] The issue at stake is here that Newton build his version of the calculus on the notion of continuing smaller and smaller cut offs when a line crosses a curve twice (a secant), whilst Leibniz introduced a special kind of infinitesimals. Numbers so small that even if you multiply them with a very large number they don’t become a “normal” number (so-called non-Archimedean) see e.g. {Bell, 2005}.

In the dotted line in case my velocity is constant. In that case I traverse equal distance in equal time. The straight line gives a more realistic picture as I chance my velocity while biking. If I blow up the picture say with 100, the shape remains the same. In the example my velocity is changing slowly and I don't' interrupt the motion by peeing behind a tree. In this case the question emerges what is my velocity at a certain time or place? Following the definition of velocity as the fraction of distance traversed and time used, we can draw some lines parallel to the x- and y-axes and measure the distance and elapsed time around say point A, which depicts the place, x1(at A) where I am at time y1. As the reader can see in the picture, the curve is not straight, because the velocity varies. The question is "what is my velocity at A (also known under the term the "event" at A). In finding out we blow-up the picture and that way reduce the measure of change until we hopefully reach a straight patch around A. In the second picture this is clarified in a case where we shrink 100 fold, but at the same time enlarge the picture 100 times. As everybody can see, we don't see much difference as the change in my velocity is quite haphazard at point A. How to handle this was the question? The way to do so is that we reach smaller and smaller entities to a point that they almost vanish, that is to say their measure come close to zero. We have no idea how to deal, not to say calculate with, so-called infinitesimals, minute, quantities.[28]



If we call the y-axis time and the x-axis distance, then F(x) is the value (function) of y at point x and F(x+h) the value a bit (h) further on.
Written as a function we have (F(x+h) - F(x)) / h , means the "value of the function F at x plus a small distance h" minus the "values of the function F at point x" divided by the small value h. The quotient of the difference on the y-axis, called $\triangle y$ (delta y) and the difference on the x-axis $\triangle x$ is called the difference quotient: $\triangle x/\triangle y$. An ever smaller distance (in meters) X divided by an ever smaller period (in seconds). If we let this differences shrink (but never reaching zero) we name this limit the derivative. The big issue is what happens

[28] Note that with a speedometer, we average over some period. I drive now 112 km/hour is mathematical sloppy language. See: https://en.wikipedia.org/wiki/Speedometer

as the value of h reaches zero. We then might get a situation that we divide zero by zero. And this is not a defined quantity. It is also this problem that Marx discusses in his mathematical works.
Working with derivatives we can define new so-called derived function. In our example biking gives rise to the velocity (change in place) function and on its turn the changes in the velocity give rise to the function representing acceleration (change in velocity). By the way this is the Newton way of picturing the problem. The ever shrinking quantity h is called an infinitesimal in the Leibniz approach. So, Newton developed a theory in which de measure of change $\Delta x/\Delta y$ is slowly going down as viewed in the picture, whilst the Leibnitz approach is taken the "differential" as dx/dy as a number. For centuries the foundational discussion was: are these two approaches ontologically the same.

4.2 Understanding the Calculus
In the 17th century the German Gottfried Wilhelm Leibniz (1646-1716) and the Englishman Isaac Newton (1643-1726) independently of each other invented schemes to handle this problem, called the calculus. The calculus is used as a tool for all problems in which we have changing parameters. [29]
The calculus is that part of analysis that enables calculations with this strange ever shrinking quality: called infinitesimals. Although in practice and in the many applications of the calculus, particularly in mechanics, as usual, the practitioners just cranked the handle, because it works, and the philosophers and theoreticians warred over the implications and differences in an ontological approach. This dispute is in some way still going on, but was 'solved' only in the 2nd half of the 19th century by a rigorous approach of understanding the notion "limit"; at the same time and after Marx and Engels discussed mathematics.
The differential calculus, or the notion of differentiation, gives us a rate of change, which as can be seen in the picture and can be interpreted as the slope (or gradient) of a straight line that just touches the curve, as pictured above. In the Leibniz notation, the smaller and smaller differences are denoted by the sign dx or dy in our case and the differential is the fraction dx/dy. Now the question is: what are these mathematical objects (dx/dy), and why can we treat them algebraically as normal numbers, though they are considered minute values, close to but not exactly zero. [30]

4.5 Infinity
Please reader, hold on, as we have to dig a bit deeper in this different approaches, otherwise it becomes completely incomprehensive why e.g. in the former USSR such strong feelings about dialectical materialist mathematics, based on Marx's struggles with the subject, became part of the catechisms. To do so, we have to introduce the quest of the infinity. Humanity knows an age old discussion on what we mean with the infinite and how we can tame this animal. This discussion is intertwined with the idea of what is a number; because if we want to count for ever, do we reach infinity? Numbers exist in various types. We have the natural numbers 0, 1,2,3,4... (in mathematics the sign" ..." 3 dots - is called ellipses, and

[29] There are plenty of elementary introductions for outsiders to the calculus, also on Youtube, just pick and choose.
[30] For a comprehensive history of the fascinating road to modern calculus see: {Boyer 1949}

means: and so forth for ever). The collection of all natural numbers is known by the sign ℕ.[31]
The extension of number systems is a historical process as people needed signs to deal with ever more elaborated calculations.

> In order to deal with problems in which sqrt -1 (square root of minus 1 or √ -1) came up, the practical solution was made to introduce complex numbers that are compositions of a normal number and a number times sqrt -1. Strange as it may look for the naive realist, these human (rigorously defined) inventions are extremely productive and again forces us to the fundamental issue of to what extent such "ideals" are real. We come back to this discussion in chpt X when we discuss Ilyenkov. See also e.g., {Courant-Robbins 1996, pp. 52-63}

As an extension of the natural numbers, we have negative numbers, which mirror the positive natural numbers around zero -1,-2, -3 ... and all these together are called integers with the sign ℤ. Integers are the normal counting devices in which negative numbers can be understood as subtraction or loss. A consequence of this mirroring is that we need the number zero, a strange thing which does not emerge from counting objects. We cannot count from say minus 2, minus 1 followed by 1, 2 .... as the step between minus 1 and plus 1 is a contrariety to all other steps in counting equal to two and not one. Therefore, zero, or nothing, is pivotal in building a consistent number system.
Subsequently, we have the rational numbers or fractions, which are natural numbers divided by other natural numbers (excluding zero) called ℚ**.** Examples are 3/4 or 137/49, needed to divide something (a cake) among various stakeholders.

As not all numbers can be expressed by fractions of integers we reach the realm of the Real numbers "ℝ" that includes also the so- called irrational numbers, which means that they are not fractions (not-ratio). The irrational class can be thought of as consisting of two branches i) the algebraic ones, which means they can be expressed in an algebraic formula such as sqrt 2, or sqrt 5. These numbers can be approximated with an endless (non-repetitive) sequence of numbers in a decimal form. E.g., sqrt 3 = 1.7320508075688772935274463... Note that this is different from a decimal representation of e.g. of the real number one third (1/3) which is a fraction (ratio) and leads in a decimal representation to reparative sequences of one or more digits, in this case 1/3= 0.333333.. . It is just a feature of the decimal system that one third cannot be written in a finite sequence of natural numbers, ii) Next to that we have the transitive numbers such as **pi** (π) which is the ration of the circumference of any circle to its diameter. There doesn't exist an algebraic expression for these types of numbers. Note that an irrational algebraic number as sqrt 2 is, can be seen in an analogue representation; simply one side (the hypotenuse) of an isosceles triangle (the diagonal of a square). Take a paper square and fold it on its diagonal.

Having made this categorisation we go back to counting. The <u>natural numbers</u> are the numbers we use in simple counting, which already indicates that they have various roles. As

---

[31] Note that "numerals" such as 43 and 666 are symbols that represent numbers. The symbol 666 symbolizes for some religions 'The Beast', whilst the symbol 43 is still waiting for redemption.

counting aids they are the abstraction of say: one bike, two bikes, three bikes, ... Importantly, these abstract counting devices can count themselves as well and then become mathematical objects in their own right: first natural number, second natural number... Henceforward, we can count all real numbers as well e.g. first irrational number, second irrational number, ... Obviously, this gives rise to nagging questions such as: if the natural numbers are clearly an infinite series, how large is this infinite compared to say the infinite amount of rational numbers or worse the reals?
Can we quantify the infinite? The mathematical breakthrough in this deep question, which kept philosophers awake for centuries, happened in the 19th century and became generally accepted in its second half. The key notion is that of the "limit", and the field in which it is developed is called set-theory, invented by the German mathematician Georg Cantor (1845-1918). Set theory is now the basis of most mathematics, and particularly in applied mathematics, used in all other sciences than mathematics per se, such as physics.

It is tempting to go into more details, but we can remain at the bottom line of the notion of a number: does it exist? Of course this is a fundamental philosophical question in which Fred Engels choked. He is (most presumably unwittingly) in line with the credo of the great Berlin mathematician Leopold Kronecker (1823 – 1891) the antagonist of Cantor, who stipulated that the natural numbers are God given, and the rest is human made (*Die ganzen Zahlen hat der liebe Gott gemacht, alles andere ist Menschenwerk*) {Weber (1893}.[32]

As FE says, in his often quoted statement: "But it is not at all true that in pure mathematics the mind deals only with its own creations and imaginations. The concepts of number and figure have not been derived from any source other than the world of reality. The ten fingers on which men learnt to count, that is, to perform the first arithmetical operation, are anything but a free creation of the mind. Counting requires not only objects that can be counted, but also the ability to exclude all properties of the objects considered" {Engels 1878, p.37.}. This statement is a direct attack on Hegel who considered mathematics as a pure human invention. Obviously, FE overlooked the fact that in the human history we know many number systems and certainly the system based on 10 was not "natural", though many pundits believed so {Ifrah, 1994, Cajori ,1928}. But he hits the nail on the head by saying that counting is comparing various objects void from anything else than just being an object. Though it is not only in contradistinction to religious notions of knowledge , as Engels insisted that notions such as number and figure are a result of human interactions with nature. In the world of numbers many strange animals can be seen.
Though this still leave many questions open. Obviously, Engels is correct if we say that pi (π) is the human representation of the ratio of the circumference of any circle to its diameter. It is a remarkable natural given that this ration is a constant, but it demands human

---

[32] In fact the natural numbers can be completely constructed from axiomatic set theory and following the rules we reach every other type of numbers such as the reals. So, historically the natural numbers emerged from counting, but then we came up with uncomprehensive notions as sqrt2 , Human ingenuity (mental labour) created a rigorous axiomatic system in which we can integrate all type of numbers.

modelling to explicate this. In more complicated mathematical objects, this semi-evident reasoning is not that easy to underwrite.
Are numbers -as the Pythagoreans say - true objects in the world and therefore all intelligent creatures must reach the same notion of number, or do their exist other ways of dealing with quantities other than land surveying and counting? Obviously the answer is yes, the problem is how.[33]
Kronecker was certainly not a Spinozist who merged the idea of a deity with the materialist world, but he was a precursor of finitistic mathematics that only accepts finite mathematical objects (such as numbers) but discards collections (sets) of all (infinite amount) numbers as a mathematical object. And just this is the basis and fully acceptable in Cantor’s set theory, to be followed by the works of, Karl Theodor Wilhelm Weierstrass (1815-1897) and Richard Dedekind (1831-1916).
The pressing point is also here; to what extent can we accept mathematical objects as real world objects, despite the fact that by introducing infinite objects and applying the consequences in our theories, we reach remarkable versatile results. As proven by set theory that, as said, is also the basis of modern theoretical physics, with all its tangible results from atomic bombs to GPS devices in cars.
Also, the inverse on the infinite kept mathematicians and philosophers alike on their toes in the discussion on the infinitesimals (the infinity small). If we can count to infinity, what about the situation of taking the reverse. What if we divide 1 (one) by an extremely large number?. Can we then still speak about a number as an entity? As it turns out we can invent such a number system based on elementary notions of counting, a point that also Lakoff and Núñez (2000) discuss in their cognitive metaphorical grounding of mathematics.

4.5 Back to Mohr

Back to Marx and the calculus, we have to stipulate what differentiation is: the technique of defining change pertaining continues functions. That means that the function, which represents the change of a variable value under the operation (influence) of another variable (such as the change in distance as a function of time), must be fluent and without sharp bends or holes. This strong demand is certainly understandable if we consider physical processes that develop normally in a smooth fashion. Abrupt situations are -in principle - not differentiable, as we cannot envision a unique straight line on the point of the abrupt change that uniquely determines a slope (the quantity dx/dy). Therefore there is a difference between the abstract mathematical notions and the applied mathematical notions in say engineering or physics. Non-differentiable, so-called monster, functions are mathematically interesting objects, but in all modern physics we stipulate that we have so-called smooth functions. This also an issue if we consider the measurement problem in some interpretations of quantum mechanics, because there at the moment of measuring, the smoothness of the expended wave function collapse into a measurable value.

---

[33] One can interpret Genesis 2:17 “But of the tree of the knowledge of good and evil, thou shalt not eat of it: for in the day that thou eatest thereof thou shalt surely die” (King James Bible) as a materialist warning that non-formal knowledge messes up abstract logic and after eating, it is only all about the metabolism of nature (e.g. an apple) and its spin-off: human thinking.

When Marx wrote his notes on the calculus, he diligently followed the traditional approaches towards the problem and -like all other students of the calculus- was wondering how to circumvent the ultimate situation where dx as well as dy become close to and even equal to zero and result in the ill-defined quantity dx/dy= 0/0. It is not the place here to review the various approaches by in particular: Giuseppe Lodovico (Joseph-Louis) Lagrange (1736-1813), Colin MacLaurin (1698-1746), Brook Tayler (1685-1731), Jean le Rond d'Alembert (1717-1783), Leonhard Euler (1707-1783), Augustin-Louis Cauchy (1789-1857) and other heroes of mathematics, whose works on the calculus Marx diligently tried to master. Marx worked on the subject as from the 50's of the 19th century, in fact the same period as the "Cantor revolution" started in Germany. As the world of pure mathematicians was very small in those years and given the British practical engineering mathematics tradition, discussions on foundation clearly did not reach Marx during his lifetime. This is also clear from the list of books Marx used {Marx, 1983, p. 274}.
Even worse, for Marx the novel idea of the limit remained terra incognita. The notion of the limit is an approach to reach but never arrive at a particular number, say zero or one. It got a strong foothold in the works of Bernard Bolzano (1781-1848), Cauchy, Weierstrass, and Dedekind. The struggle of Marx is excellently explained by {Struik 1948c}, who received already in 1935 a typescript from the original German work {Struik 1948c, p.183, footnote 7}, which he used for this paper. He also wrote an article for the 1933 first Russian edition (see section 5 below), in Russian. Most presumably his 1948c paper is a sequel to his Russian paper. It is equally a pity as remarkably that Struik, as expert in differential geometry, never came back to his survey of Marx's notes in order to confront his conclusions that Marx aim was to consider the "symbolic differential coefficient" dx/dy as a starting point. It suggests that we have to take change as prior to rest.[34]
Marx struggles with the ill-divined notion of *dx/dy*= 0/0. He "solves" this by declaring that dx/dy is of operational or symbolic form and the limit must become zero {Marx 1963, Struik, 1948c, p. 193}. As Struik correctly notes: "Marx wanted a method which actually followed the process variation of the variable and in this process itself defined the derivative 0/0, in which case it can be endowed with a new symbol *dy/dx"* {p.190}. The issue is how do you reach motion out of rest and as dx/dy is a symbol for motion, it *sublate* rest, hence it is suggested that 0/0, which is ill-defined, can be seen as a new notion: motion out of numerical rest.
On a more general level it is again the key quest of how we approach reality (in this case motion) with the help of discrete notions such as numbers, which is a central "challenge" of human knowledge. Biologically, we simply can only calculate in quantities, which is one of the reasons Hegel after taking being (*Wesen*) as starting point elaborates quality which is a breakdown of quality.[35]

[34] I don't read Russian, but I take the Russian paper knows the necessary pledges to the Diamat flag, which don't appear in the 1948c paper

[35] Although Hegel does not ground his teaching of logic on the limits of the human body and its limitations.

Already said before, if we think of a circle, it only becomes an operable object in terms of a coordinate system. [36]
By suggesting that numbers can metaphorically be seen as points on a line (like an ordinary ruler), we can think of the number line as a "dense" collection of ever more close neighbouring points. "Weierstrass settles the question of the *existence* of a limit of a convergent sequence by making the sequence (...) itself the number or limit" {Boyer, 1949, p.286}. "Previous writers generally had defined a variable as a quantity of magnitude which is not constant; but since the time of Weierstrass it has been recognized that the ideas of a variable and limit are not essentially phoronomic (relating to motion, considered without reference to force or mass), but involve purely static considerations. Weierstrass interpreted a variable x as simply a letter designating any one of a collection of numerical values. A continuous variable was likewise defined in terms of static considerations" {ibid}. Boyer concludes in his *History of the calculus*:
"In retrospect, it is pertinent to remark that whereas the idea of variability had been banned from Greek mathematics because it led to Zeno's paradoxes, it was precisely this concept which, revived in the late Middle Ages and represented geometrically, led in the seventeenth century to the calculus. Nevertheless, as the culmination of almost two centuries of discussion as to the basis of the new analysis, the very aspect which has led to its rise was in a sense again excluded from mathematics with the so-called "static" theory of the variable which Weierstrass had developed. The variable does not represent a progressive passage through all the values in the interval. Our vague intuition of motion, although remarkably fruitful in having suggested the investigations which produced the calculus, was found, in the light of further elaborations in thought, to be quite inadequate and misleading".{ibid p 288}.
The persevering reader, who might at this point become a bit dizzy, realises, I hope, that in pure mathematics we see a pendulum motion with relation the notion to fixed valued numbers and variable quantities. This keeps the fundamental (physical) notion of motion in limbo.
The lessons learned are that it is, at least in this field of human investigations, impossible to extract existential truth values from mathematical modelling. It teaches us that intuitive ideas can be casted in formal systems, such as set theory and complex numbers, which leads to the remarkable successes from Alternating Current coming from sockets in the kitchen wall to using your GPS to find your way to the neighbours, thereby leaving the intuitive notions to colloquial speech. Obviously, the pragmatist will see proof of his philosophy, but for the materialist's worldview we are facing a nice challenge.
It goes without saying that in the applied sphere; these kind of foundational discursions are normally smeared out by introducing necessary auxiliary axioms or simply by approximations. The foundational discussions are certainly not finished at all.[37]

---

[36] In applied mathematics, e.g., Theoretical physics and in particular differential geometry, starting with set theory and developing manifold theory, we develop a whole skyscraper, based on – on can say building regulations- such as having a topology, a vector space, an affine connection as well as a metric, etc. Here the derivative is a vector tangential ('orthogonal') to another vector.

[37] On the issue of the debate about the infinite and the related concept of continuity over the centuries, I highly recommend the reader Moore (1990) and for an excellent overview with a more

Interestingly, in 1960 the American mathematician Abraham Robinson (1918-1974) developed, fully based on standard mathematical logic, so-called Non-standard analysis in which a self-consistent theory enables infinite and infinitesimal numbers, also named hyperreal numbers. No dialectics at all. For Robinson's unexpected success in Mao's China; see below section 9.3.6.
Having said all that, it is time to follow the trajectory of the road to publication of Marx's about thousand loose manuscript pages and more importantly what Marx's considerations were, and on top of that how in de DIAMAT tradition his thoughts got superhuman value. However, historically it is that firstly Marx's other writings were consecrated and after finding the manuscripts, by proxy, his mathematical trials became also of foundational value. We will follow this with the much dismissed writings of Engels on mathematics, and how his evangelists do not frame these comments in their time and polemical character, but as icons of socialism. Again, for me it is not how well Marx and Engels could do and did their homework, but how to deal with the concepts mutually determining entities or concepts and the notion of change and dialectics.

5.
The bumpy road to publication of Marx's mathematical manuscripts

The first public announcement of the existence of Marx' notes on mathematics, was by Ernest Kolman[38] (1892-1979) in a presentation to the 1931, 2nd International Congress of Science and Technology in London. The Russian delegation stood under the chairmanship of N.I. Bukharin (1888-1938), already then on his way out of the leadership of the Communist Party. As the arrival of a Russian delegation was totally unexpected, there fitting in to the programme was problematic. Nevertheless, the Russian presentations in particular that of the physicist and historian of science Boris Gessen (or Hessen in the west) called "The social and economic roots of Newton's 'Principia'" and Bukharin's "Theory and practice from the standpoint of dialectal materialism" created a storm of enthusiasm among left wing activists and scholars, initiating the strong British tradition in the field of the history and sociology of science. A vivid description is given by Werskey {Werskey, 1988 chapter 5.1, 'A Russian Roadshow', pp. 138-148}. A recent deep study on the difficulties the Russian delegation encountered is given by {Chilvers, 2015, with unique pictures}. This was the more remarkable as the entire framework of the USSR academic as well as universities and research institutions were in a turmoil in the regime's policy to direct research to applied work and trying to replace scholars by "Red professors" {see e.g.: Jarovky (1961), Graham (1987), Bailes (1978)}. Interestingly Kolman, as he told Graham in the 1970s, had been the CP party secretary for the delegation, and was thus responsible for party discipline {Graham, 1985}. Gordin (2017) remarks: the sinister person Kolman "... was practically the only member to survive to the end of World War II. Every other member

philosophical-mathematical approach Bell (2005, 2017). The standard history of the calculus is written by Boyer (1949)].

[38] The name of Kolman known many spellings in the literature, we follow the spelling used in the paper quoted.

(besides Ioffe) was executed in the purges or died in a prison camp". More on this enigmatic person, below in section 9.3.2.
The presentations of the USSR delegation where immediately translated and published by the (we would now say pop-up) publishing house Kniga (book in Russian), and therefore became a source book for philosophers and historians of science since {Kniga, 1931}. Kolman (his name is spelled as Colman in the book) has three relatively short presentations: "Dynamic and statistical regularities in physics and biology" (12 pp), "The present crisis in the mathematical science & general outline for their reconstruction". Both will be dealt with below if we discuss DIAMAT maths, and a third one: "Short communication on the unpublished writings of Karl Marx dealing with mathematics, the natural sciences, technology, and the history of these subjects (3pp). The announcement is a simple list of unpublished works of Marx on various subjects. No mentioning is made about plans for publication.
According to Kolman {Marx 1983, p.255}, already in 1927 Emil Julius Gumbel (1891-1966), the German mathematician and anti-fascist activist, published a report on the manuscripts in Russian.[39]
As Kolman says "In 1931, with the appointment of the well-known activist of the Bolshevik party V. V. Adoratsky to be director of the (Karl-Marx-Friedrich Engels) Institute (in Moscow), work on the manuscripts was given a new direction.[40] As head of the Marx Study Centre at that time, I was acquainted with the transcribed portion of the manuscripts and with the preparatory work towards their publication, and I was convinced that E. Gumbel was unable to appreciate completely either the importance of their publication or their philosophical- mathematical significance. At my suggestion, the board of directors of the institute enlisted for the work on the manuscripts S.A. Yanovskaya, leading a team which was joined by the mathematicians D.A. Raikov and A.I. Nakhimovakaya".
Sofya Aleksandrovna Yanovskaya Neimark (1896-1966), who edited the Marx's manuscripts was originally a "red professor" and party functionary, who became an important logician on her own merits {Bazhanov, 2001, Anellis, 1996}. Contradicting Bazhanov, who writes "unlike E. Kolman -Yanovskaya's co-author at that time- Yanovskaya, never wrote direct or ideological denunciations"[41], the mathematician Lorentz (2002) in his personal recollections of the role of mathematics and politics states "The main anti-Lusin lecture at MGU (Moscow state university) was delivered by Mrs. Yanovskaya".[42]

[39] Gumbel's interesting book *Vier Jahre Politischer Mord* (1922) dealing with the period 1919-1922, includes lists of people killed { http://www.gutenberg.org/ebooks/39667 }. In 1932 his house was ransacked and he was among the first cohort of 33 persons whose German citizenship was taken away by the Hitler government (Ausbürgerungsliste des Deutschen Reichs von 1933). He became a researcher at Colombia University, NY.

[40] The appointment of Vladimir Viktorovich Adoratsky (1878–1945), followed the disgraceful arrest of the revolutionary old timer David Borisovich Goldendakh (Riazanov) based on a testimony, under torture, by a collaborator that he was 'helping Menshevik counter-revolutionary activity'.

[41] We talk about the disgraceful "Lusin affair" in 1936 where religious mathematicians of international standing were sued {Graham & Kantor, 2009}.

[42] The dubious role of Yanovskaya, is clear from the recent publication {Demidov et. al. 2016} which give a complete story including minutes of meetings. Anellis with his particular ideas

This small detour is warranted to understand the circumstances in which Marx's mathematical work was prepared for publication and catapulted to a status of utmost importance. In 1933, fifty years after the death of Marx, parts of these manuscripts, including Marx's reflections on the essentials of the differential calculus, which he had summarised for Engels in 1881 in two manuscripts accompanied by preparatory material, were published in a Russian translation, the first in the journal *Under the Banner of Marxism* (1933, no. I, pp. 15-73) and the second in the collection *Marxism and Science* (1933, pp. 5-61). However, even these parts of the mathematical manuscripts have not been published in the original languages until 1968. "Although Marx's own work, naturally, is separated from the outlines and long passages quoting the works of others, a full understanding of Marx's thought requires frequent acquaintance with his surveys of the literature. Only from the entire book, therefore, can a true presentation of the contents of Marx's mathematical writings be made complete" {Yanovskaya, 1968}.
This is certainly true, but it avoids the issue of a comparison with the contemporary modern mathematics, that developed in the same period.
In 1968 a bilingual publication German- Russian was published in Moscow. The first part, not containing all Marx's extracts and comments of the books he read, was published in English in 1983 {Marx 1983}. This volume has as appendices, extracts from some letters to and from Marx and Engels, and some articles such as Yanovskaya's review of the manuscripts and a paper by her and Kolman on Hegel. A second complete English translation from the Russian version has been published in 1994 {Marx, 1994} edited by Pardip Baksi, stating: "However, the preface, editorial comments, notes and appendices are all in Russian only. Hence, Russian is the only single language through which the entirety of this volume becomes accessible. The present translation has throughout followed the texts, comments, notes and appendices in Russian". This volume knows ten contributions by scholars as supplements. It also includes a special essay by the editor.[43]

As said above, Marx diligently tried to understand the foundations of the calculus and failed together with all the famous mathematicians he quotes. It is interesting to see that in "Marxist" literature Marx's valid critique is echoed as proof of the non-dialectical nature of the "old" calculus. The last victim is Carchedi (2011) in his interesting attack on the non-dialectical behaviour of his fellow economists who study Marx's reproduction schemes (see also below section 9.2. The point is that a dialectical solution to the problem defining dialectics seen as "logic of contradictions" did not work out. The notion of variation (changing values), as said above, was replaced in the 2nd half of the 19th c. by the notion of stasis by the great arithmetizers Cantor, Dedekind and Weierstrass, leaving the

---

of Stalinism (see below section: 9.1: Van Heijenoort) refrained from mentioning this affair in all his publications.

[43] Although scheduled as: MEGA2, Ab. I Bd. 28: Karl Marx: Mathematische Manuskripte (1878-1881) and Ab. IV Bd. 30: Mathematische Exzerpte aus den Jahren 1863, 1878 und 1881 (insbes. Trigonometrie, Algebra und Differentialrechnung), no publication dates are yet known. A draft paper {Alcouffe and Wells, 2009} by members of the MEGA2 editorial board circulates, but deals mainly with probability and statistics.

philosophical issue of continuity to the dogs. This means serious homework for dialecticians as the reduction of a notion of change as being a new stage in the confrontation between antagonist forces remains an open research programme. Lakoff and Núñez see this conceptualizing of naturally continuous space in terms of discrete entities and holistic motion in terms of stasis and discreteness, as proof for their mechanical neurological models {Lakoff and Núñez, 2000, p. 293}.

As already said above this highly successful discretization programme, which is also at the basis of digital computing and digital modelling of everything under the sun, is asking for a fresh look. To what extent can be approximate motion by stasis?[44]

6.
Hegel and mathematics

not in de mood yet. Mainly a structuring existing maths into his dialectical scheme, in *Logic*. Nothing on how his scheme might give rise to novel insights.

7.
Engels and mathematics
7.1 Engels and Math

"It goes without saying that my recapitulation of mathematics and the natural sciences was undertaken in order to convince myself also in detail—of what in general I was not in doubt—that in nature, amid the welter of innumerable changes, the same dialectical laws of motion force their way through as those which in history govern the apparent fortuitousness of events; the same laws which similarly form the thread running through the history of the development of human thought and gradually rise to consciousness in thinking man; the laws which Hegel first developed in all-embracing but mystic form, and which we made it one of our aims to strip of this mystic form and to bring clearly before the mind in their complete simplicity and universality. It goes without saying that the old philosophy of nature—in spite of its real value and the many fruitful seeds it contained —was unable to satisfy us. As is more fully brought out in this book, natural philosophy, particularly in the Hegelian form, erred because it did not concede to nature any development in time, any "succession", but only "co-existence". This was on the one hand grounded in the Hegelian system itself, which ascribed historical evolution only to the "spirit", but on the other hand was also due to the whole state of the natural sciences in that period". {Engels 1878, 2rd preface 23 sept 1885, MECW V25, p.12}.

---

[44] Preluding the discussion on Diamat and mathematics it is interesting to note that the founder of the so-called intuitionistic school of Logic, the Dutchman Luitzen Egbertus Jan (Bertus) Brouwer, 1881-1966) who was despised by the official Stalinist railing machine (but at the same time an important teacher of famous Soviet mathematicians), took the phenomenon of temporal variation as fundamental {see J.L. Bell 2005, p.321 and Van Stigt, 1990}. To make things even more complicated one may consider intuition as a bodily (hence not classical idealistic) phenomenon, as a kind of sense, which can give a different turn on the contempt for the intuitionistic school by the materialist USSR style.

The important notion here is the distance taken from “oppositions” towards the notion of becoming, the ever evolving dynamics of the real world.

As already said above, the AD book is a polemic work and only its third part -socialism- which became a popular publication in its own right, is a programmatic study towards socialist politics. The main idea is to show that in all field of human enquiry, we deal with a dynamic interplay of objectified notions. Notions which became linguistic currency within a particular socio-historical context. The best example is the notion of ‘working class’. Throughout history we have seen repression of humans by humans in a great variety of form. Though in 19th century there was some indication and hope that in a primitive society a greater level of equality existed. This induced the idea of UR-communism and the, Christian inspired, hope for a return of that pristine phase of humankind of a successful socialist revolution.[45]

In Engels’ -for his time extraordinary- book *The Origin of the Family, Private Property and the State. In the Light of the Researches by Lewis H. Morgan* {Engels, 1884, pp. 129-257} based on the then to him available anthropological knowledge a first attempt is given about the role of private ownership of the means of production. In showing how society and culture developed in function of the technical progress of the means of production vis-a-vis the human creativity expressed in human labour power, the capacity to conscientiously produce, is projected to the possibility of better future for humankind. In same way as Marx in *Capital* the ability to produce surplus production is the driving force for human emancipation. As is the foundation of Marx’s analysis of the period of the capitalist mode of production, a ‘working class’ which doesn’t have independent access of the means of production emerges together with its antagonist class of ‘capitalists’ that do own these means of production. Both existences are fully co-determined. It goes without saying that such a categorisation, like every categorisation, has limits in its historical dynamics. Labour historians investigate into minute detail the great variety of repression and exploitation, which they call subaltern labour, within the emerging as well as mature capitalist mode of production {van der Linden, 2008}. This research shows that there is always a tension between a model (two antagonistic classes and nothing in between or around them) and the muddy reality from which such a model is extracted. People often forget that also in all fields of natural science and biology terms and categories have changing meaning as well, but their formal definitions can stand out longer. Defining the working class as those workers that have to sell on the market their labour power for money because they don’t have the means of production to survive is a strict definition. Discussions arise if we have segments in society who next to their “capitalist exploitation as labourer” have other means of income such as tilling their garden or having small micro companies themselves in which this precarious labour is tinsel in the neo-liberal dream palace of self-employed people. In chemistry we can define an atom as the smallest entity of a chemical element. But also there we have variation called isotopes, which under certain circumstances behave differently

[45] [In his monumental series *On Marxism and Theology*, Roland Boer discusses this theme extensively. See in particular volume 5: *In the vale of Tears*, 2014, chapter 2; ‘Myth’ pp. 69-124.

from each other. Molecules we can define as the smallest entity of a chemical substance, although optical chirality (mirror images) can determine (bio)chemical reactions. Here, and in a plenitude of other examples we see that a firmly defined object, can develop its meaning in the course of history. Also terms can change meaning or have to be split into new words. Concepts as force, energy and momentum took a long time to become strictly defined entities.[46]

In mathematics, life is a bit different as here, in axiomatic systems such as Euclidian geometry, definitions are pertinent and new objects get new definitions. Contrary to the fluidity in naming objects and interactions in the natural sciences, where the content of the name (e.g. atom) may shift, in mathematics the rules are far more abstract and strict. However, this does not mean that in every sub-field or every mathematician, adheres, as a robot, to the same names and definitions. A result of this is that next to mathematics we know the fully axiomatic systems of logic and the relationship between logic and mathematics is an ever hotly debated issue. Hence, we are confronted with two aspects of Engels' dealing with the matter of dialectics of nature.

On the one hand he, correctly, emphasises the dynamical changes in natural science as interplay between ever novel experimenting and theory building. A development that was and is grounded in the socio-economic developments of society, as well as in the independent phantasies of individuals who often extracted their -sometimes mind-boggling ideas- from the contemporary ideologies, religions, etc. This -we call it- historical materialist - approach certainly sees mutually interactive or dialectical characteristics and became the root of the various sociology of science schools.

All natural sciences try to describe and understand the mutually interacting objects often in terms of attractive and repulsive forces, as is stipulated in Newton's third law which says that: For every action, there is an equal and opposite reaction.

The second aspect is to formalise matters in what is sometimes called a dialectical logic. Naming a positive quality antagonistic to a negative one (atomic nucleus and electron, or two versus minus two) is not very productive as they don't necessarily compose a 'totality'. It is very formalistic to say that the unity of an electron and a positron in the light flash they create in meeting each other is dialectical. Where the working class cannot exist without the class of capitalists, they are co-defined; there is no reason to co-define a positive and a negative electric quality. Electrons can happily live on their own. We defined the electron and subsequently quantum mechanics suggested the positron as an electron with positive charge, which was measured for the first time in 1932. Though, if we adhere to quantum field theory we can imagine that the void is build-up of 'virtual' positive and negative particles. It would be a very mechanical idea to name the void a dialectical unity of positive and negative virtual particles, as dialectics demand a subsequent negation of that very void[47].

---

[46] For an historical overview of the notion force see: Jammer, 1999. For a study of the formal establishing of the notions, energy, force and matter in the 19c see: Harman, 1982.

[47] In many examples of perceived dialectics in physics, they don't go beyond simple oppositions often induced by symmetry considerations. That way the claim that scientists are already dialecticians whiteout knowing it is as easy as useless.

Engels' lacks of knowledge of the latest and hottest developments in mathematics, mainly on the continent, is understandable. A bit too enthusiastically he took the current open questions as proof for an idealistic deformation of mathematics. Though Engels' own knowledge of contemporary maths was scanty. He was convinced Hegel's dialectics already solved many questions.
Read for instance his contemptuously remark in the 2nd preface of AD:
"In this connection there was only one unrecognised genius of a mathematician who complained in a letter to Marx that I had made a wanton attack upon the honour of -1^1/2 It goes without saying that my recapitulation of mathematics and the natural sciences was undertaken in order to convince myself also in detail—of what in general I was not in doubt—that in nature, amid the welter of innumerable changes, the same dialectical laws of motion force their way through as those which in history govern the apparent fortuitousness of events; the same laws which similarly form the thread running through the history of the development of human thought and gradually rise to consciousness in thinking man; the laws which Hegel first developed in all-embracing but mystic form, and which we made it one of our aims to strip of this mystic form and to bring clearly before the mind in their complete simplicity and universality". {Engels 1878- AD Vol 25 p. 12,}.[48]
Engels' conception of mathematics was already old hat when he wrote AD & DoN.
He takes a naturalistic point of view and tries to defend that all mathematics is already grounded in the real material world. For that reason he misses important issues such as the imaginary (complex) numbers based on the idea of the square root of -1, the novel notion of the limit, the notion of higher dimensions as well as non-Euclidean spaces, etc. He writes e.g*:* "In its operations with variable quantities mathematics itself enters the field of dialectics, and it is significant that it was a dialectical philosopher, Descartes, who introduced this advance". {Engels 1878, AD p 112}.
Which means that the great brake trough of abolishing "old fashion" motion to numerical stasis (see above) was unknown to him and hence the philosophical discussions and critique on these ground-breaking developments are not dealt with.
He totally misses the point that even if abstract knowledge or models are the result of more elementary practical issues, this does not make them voodoo.
Notorious is Engels notion for his notion of 'mirror image' (Abbilder, Widerspiegelung), the idea that human thoughts, hence mathematics, are more or less one-to-one representations of the material reality outside the skull. Engels writes: "Pure mathematics deals with the space forms and quantity relations of the real world—that is, with material which is very real indeed. The fact that this material appears in an extremely abstract form can only superficially conceal its origin from the external world". {Engels 1878, AD p37}. And:

---

[48] Heinrich Wilhelm Fabian was a German living in the USA. He wrote Marx on Engels misinterpretation of sqr -1. Engels mentions him in various letter: e.g. to Kautsky on 11 April 1884: *After that he went for my dialectical approach to mathematics and complained to Marx that I had defamed sqr -1 now the fun is beginning all over again.* (MECW V47 p. 72) or the same complaint about Fabian in a letter to Adolph Sorge {MECW V47 p 295}. It is clear that nor the recipients nor later readers felt the need or had the knowledge to point to Engels mistake in understanding. See also 9.1 Van Heijenoort below.

“But, as in every department of thought, at a certain stage of development the laws, which were abstracted from the real world, become divorced from the real world, and are set up against it as something independent, as laws coming from outside, to which the world has to conform. That is how things happened in society and in the state, and in this way, and not otherwise, pure mathematics was subsequently applied to the world, although it is borrowed from this same world and represents only one part of its forms of interconnection—and it is only just because of this that it can be applied at all”. {idem}.

This type of plain simplifications plaid havoc in Stalinist science, as it denies any free mathematical modelling.
Engels conflates physical laws with mathematical laws, which denies the fundamental difference between physical theory building, based on experimental results and the free creations of the mind. For instance if we experience that in two dimensions of a flat space, the sum of the angles of a triangle are always the same, named 180 degrees, we can invert this into an axiom, in this case of Euclidian geometry. The research on non-Euclidean geometry (curved space) took off in the early 19th c. as an intellectual game. Only in the next century, in General Relativity it turned out that this is a physical reality and now 100 years later, everybody is using a GPS driving an automobile[49]. Hence Engels equating the mathematical notion of the 4th dimension with Spiritism because the Leipzig professor Johann Karl Friedrich Zöllner (1834 1882) turned -like many intellectuals in the 2nd half of the 19th c.- to Spiritism, is plain stupid. The 4th dimension is not wrong because Spiritism is nonsense, nor because we live in a three dimensional material world. {Engels 1873-1883) DoN p352}. The reverse is true, higher dimensions are products of human bodily (material) thinking and are versatile tools in understanding and engineering our 3D (space) and 1D (time) world. It is funny that at present the latest investigations in black holes and gravitational waves are explained to the general public as ripples in the “fabric” of ‘four dimensional space-time’, as if the model of a rubber sheet is a reality.
In the same way, Engels does not understand the rest of novel mathematics of his time (see below the section on Van Heijenoort 9.1). In itself this is not a drama; it “only” means that Engels use of historically discarded maths cannot any more of use in a theory of the Dialectics of Nature. The fact that the Stalinist mathematician (for Kolman see above and others see below) struggles with maths and the holy trinity of dialectical laws is of historical importance and gives us arguments not to enter this *cul de sac*. But as with Engels fallacy with the example of Spiritism, it is <u>not</u> a sufficient argument to discard any notion of dialectics in nature. Even if we reduce the notion of dialectics to sociology and history, still in that case our (dialectical) thinking is a material activity of the brain and part of nature.

7.2 Where is maths coming from?

[49] The interesting issue is that a GPS system makes use of both the, opposite, effects of the measuring of time in special relativity theory and general relativity theory, whilst its electronics needed to calculate everything is based on quantum mechanics which is within the realm of special relativity but ontologically not in that of the general theory. This enigma is the source of the present research in e.g. string theory.

First of all, even if we start with a genetic material human grounding of counting and understanding small collections of objects the way, such as Engels suggests and Lakoff and Núñez {Lakoff and Núñez, 2000} tie this to motoric and neurological investigations, it does not solve the question. After all, we don't have a clear notion yet of what we inhered. Engels was wrong when he leaned over to simplistic Lamarckism when writes the notes:
"If, for instance, among us the mathematical axioms seem self-evident to every eight-year-old child, and in no need of proof from experience, this is solely the result of "accumulated inheritance". It would be difficult to teach them by a proof to a bushman or Australian Negro". {Engels 1873-1883, DoN p. 545} and "Two kinds of experience — external, material, and internal— laws of thought and forms of thought. Forms of thought also partly inherited by development self-evidence, for instance, of mathematical axioms for Europeans, certainly not for Bushmen and Australian Negroes" {idem p. 596}. But this wrongness, which was staple knowledge in his time, just as Spiritism, does not invalidates Engels "research programme" in de relation of dialectics and nature. So, the idea that, evolutionary, our species must have developed an idea of number or amount in order to survive and develop is acceptable. But to locate this notion in an active brain activity, represented in neural activity (the only activity we can measure today) is, I think, a bit too simple as well.
Obviously, all (mathematical) thinking is a material brain activity, and many mathematical ideas and intuitions come to the fore by physical experiences and metaphors. But the beauty is that this is not a mechanical affair. Even if we shift from the Newtonian mathematical model to the Einsteinian, nothing has changed in the two mathematical constructs. Experience changes the models used to describe phenomena. All human senses are based in matter and also thinking is a bodily activity, it stops when you die. Innate -evolutionary grown capacity of counting- might ground our ability to invent mathematics, and Lakoff and Núñez suggest this is done by what they call conceptual metaphors" "Conceptual metaphor is a cognitive mechanism for allowing us to reason about one kind of thing as if were another. This means that metaphor is not simply a linguistic phenomenon, a mere figure of speech. Rather it is a cognitive mechanism that belongs to the realm of thought .... technical meaning: It is a grounded, inference-preserving cross domain mapping - a neutral mechanism that allow us to us the inferential structure of one conceptual domain (say geometry) to reason about another (say arithmetic). Such conceptual metaphors allow us to apply what we know about one branch of mathematics in order to reason about another branch" { Lakoff and Núñez, 2000, p. 6}
This suggesting is most interesting because it indeed grounds abstract thinking in neural activity. But neural activity is -at present- not more than a descriptive experimental science where we can measure ("see") how mental processes activate neural matter and how neural matter communicates among neural matter. The flimsy references Lakoff and Núñez give to motoric and neurological investigations, point all to computational modelling.[50]

---

[50] E.g. their claim about the PhD thesis of S.S. Narayanan that should tell us that the same neural structure used in the control of complex motor schemas can also be used to reason about events and actions. But if we look up this thesis at: we see that this work: *Knowledge based Action Representations for Metaphor and Aspect KARMA,* is about computer modelling. http://www1.icsi.berkeley.edu/~snarayan/publications.html This means that Narayanan was able to

Cognitive science and artificial intelligence are dynamic fields which try to merge fascinating measurements of neural activity with algorithmic modelling. There is no theory at all; it is brilliant pragmatism, which opens new vistas of knowledge.
Interestingly, we are back to the old discussion about the difference between mechanical materialism and philosophical materialism: the central issue of Lenin's critique on the Emperio-critisists {Lenin, 2008} and well elucidated by Dominique Lecourt {Lecourt, 1973} and Evald Ilyenkov {Ilyenkov 1979}. Following Engels intentions, we owe Lenin new criticism on the tension of abstractions made by bodily activities and the social reality of those abstractions.

8.
Lenin and mathematics

Although all Diamat literature is naming Vladimir Ilyich Ulyanov (1870-1924) in one and the same line as Marx and Engels on all subjects, Lenin did not spend time on abstract mathematics. However, like any serious researcher in economy Lenin was well versed in statistics. Kotz and Seneta, { Kotz and Seneta ,1990, and references therein} try and make a scholarly inventory of Lenin's work and references as a statistician and his role running statistical agencies in the USSR. As statistics is a *condition sine qua non* for a planned economy, Lenin's 'genius' exemplified in extreme adulatory publications is reviewed (on 149 items in Lenin's work, people found references to statistics). The authors provide an historical overview of early Russian statistic research and Lenin's work on grouping (e.g., is the peasantry grouped according to their legal situation, farm size, number of draught animals, ...) which is essential to understand the social composition of the peasantry. They stress the great emphasise Lenin puts on the integrity of the data and compare this with Stalinist statistics, were statisticians were simply shot if the figures turned out undesirable {Kotz and Seneta ,1990, p.90}.
Lenin's 100th birthday obviously showed an avalanche of adulatory assessments on all his writings, including mathematics. It will always remain a secret to what extent the authors themselves took their writings seriously. The well-known mathematician and expert in probability theory Boris Valdimirovich Gnedenko (1912-1985) published a paper {Gnedenko, 1970} in which he states: "In Lenin's enormous literary legacy there is not a single work devoted to mathematics. And none the less it is difficult to overestimate his influence on the content and progress of soviet mathematics". The innocent reader with some knowledge of the Stalinist culture can only agree that indeed, despite Lenin's absence in maths, the party ideologues used every opportunity to use Lenin in curbing so-called idealistic mathematical excursions.
Gnedenko then mainly deals with Lenin's theory of reflection as written down is his *Materialism and empiro-criticism"* and repeats the idea : "We know that, in fact, not a single mathematical concept, not a single mathematical theory originates in free creative power, without any connection with social problems, with practical matters, with the development in science". A strange genuflection to applied science as basis for all creative knowledge. Obviously the fact that Georg Cantor, the genius who tackled the abstract notion

---

model neurological activity and that does not mean that from this success we can claim that we are talking about the same processes. No, as Lakoff must know, it is a metaphor!

of the (trans) infinite in his set theory and was so impressed with his results that he wrote pope Leo XIII that his theory could prove the infinity of god {Dauben, 1977}, still is the result of Cantor's brain waves rolling through the material neurons in his skull.
An even more famous mathematician and party member, A.D. Aleksandrov (see also below section 9.3.4.) spent a long essay on mathematics and dialectics for the 100 years Lenin festivities.

9.
What do ‘Communist’ mathematicians say about Marx’s manuscripts and Engels?

> [I use the term communist in an irresponsible sloppy way, as it includes the Stalinist zealots and inquisitors, who denied the essence of communism: the equality of humans in strive for self-organization]

9.1. Jean van Heijenoort

Jean Louis Maxime van Heijenoort (1912 – 1986) was one of Trotsky’s assistants and became an important mathematician after he left Trotsky in Mexico in 1939 to help the American Trotskyites and subsequently started to study mathematics in the US. He became a noted academic historian and specialist in mathematical Logic.[51]
In the US, Van Heijenoort became a member of the “Trotskyite” Socialist Workers Party (SWP). Within this party and in confrontations with the breakaway Workers Party, fierce discussions took place about the prospects of a revolutionary workers upswing at the end of the war, as happened after WW1. Also the discussion on the nature of the Soviet Union was central. To what extent was it possible to believe that the SU was still a progressive country (even if the means of production were not in the hand of individual capitalists)? Van Heijenoort was very pessimistic, felt betrayed, and wrote:
“A century [the Communist Manifesto was published in 1848-jk] now lies behind us, and experience has turned in its verdict. The political capacity of the working classes has revealed itself as a never-ending capacity for being "betrayed." … Stalinism is after all only the most monstrous link in a chain of bankruptcies. … The end of the Second World War, out of which no movement emerged to indicate that the proletariat was yet fit for power, has, I believe, conclusively invalidated the fundamental hypothesis of Marx”. {JeanVannier, 1948, in *Partisan Review* as quoted by Le Blanc, 2018, full text in Van Heijenoort 1948.}

Van Heijenoort’s many bitter experiences with Stalinism, internal political struggles, and the beginning of the Cold War, prompted him to write in the same year 1948, an angry but scholarly attack on the mathematics of Engels. Proving that the canonization of Engels’ writings in *AD* & *DoN* on mathematics, was based on mathematical ignorance and Hegelian dogmatism. {Friedrich Engels and Mathematics (1948) in: Jean van Heijenoort: 1985, pp. 123- 151}

---

[51] For bibliographic information see {Van Heijenoort, 1978} and his biography by the wife of his close mathematical colleague {Feferman, 1993}. Irving H. Anellis, who’s PhD in logic was supervised by Van Heijenoort published various papers on the Van Heijenoort’s mathematical work. He also wrote a review of {Van Heijenoort 1978} in {Anellis, 1979} which led to a fall out between both logicians, as Anellis suggested that Van Heijenoort underwrote the thesis that Stalinism was a natural consequence of Bolshevism. In an extensive self-defence Anellis explicate his ‘formal logical’ understanding of Stalinism and his negative opinion of Trotsky in detail. {Anellis, 1988}. In 2012 a special issue of *Logica Universalis* on Van Heijenoort has been published {Anellis, 2012}.

Van Heijenoort follows the literature Engels used and shows that Engels was totally unaware of the massive steps forward the field made at the same time of writing up his ideas on maths, as well as that Engels mixes physics laws with mathematical thinking.[52]
It would be too much to review all this, as it painstakingly shows that Engels naturalistic ideas of mathematics have nothing to do with the developments within the field.
But the main point Van Heijenoort makes is of course not that Engels was a dilettante in the field, but that he used his limited knowledge in declaring dialectical oppositions which, and this is the great drama, remain ingredients for generations to follow and try to master dialectical thinking.
As an example: Engels writes: “In a given problem, for example, I have two variables, x and y, (...) I differentiate x and y (...) And now, what have I done but negate x and y (...)? In place of x and y;, therefore, I have their negation, dx and dy, in the formulas or equations before me. I continue then to operate with these formulas, treating dx and dy as quantities which are real, though subject to certain exceptional laws, and at a certain point I negate the negation, i.e., I integrate the differential formula, and in place of dx and dy again get the real quantities x and y, and am then not where I was at the beginning, but by using this method I have solved the problem on which ordinary geometry and algebra might perhaps have broken their jaws in vain.” {Engels 1878, *AD p. 128}.*
Van Heijenoort comments: “In these two (the first is on Engels’ dealing with the sqr of minus one -jk) examples 'to negate' means four different operations: (1) to multiply by - 1, (2) to square a negative number, (3) to differentiate, (4) to integrate. What is the common feature of these operations that would allow Engels to subsume them under the concept of negation? A few pages later he tells us that 'in the infinitesimal calculus it is negated otherwise than in the formation of positive powers from negative roots’. But he never gives us the slightest hint as to what distinguishes the four 'negating' operations from other mathematical operations. Or can any mathematical operation be considered as a 'negation'? Then, what does the 'negation of the negation' mean? It is both impossible and useless to criticize Engels' use of this formless notion in the field of mathematics”. {Van Heijenoort 1985 (1948), pp. 146-147}.
With this devastating critique on Engels and the, above discussed, limitations of Marx’s mathematical knowledge we are back on square one (a modern mathematician would say square zero, as the first step has still to be made).
The platitudes so common in Stalinist culture based on the canonization of both Engels’s *Anti-Dühring* and his notes published as *Dialectics of Nature*, pose two questions for contemporary Marxists. 1) How dealt eminent Stalinist Mathematicians with this issue and 2) how to we crawl back to the original problem of transcending tautological reasoning such as in modern logic and mathematics and try and find models that go beyond this highly successful approaches.
In nature and society we experience mutually influencing “forces” that together form some kind of totality. Interactions give rise to novel situations; we call this the dynamics of dialectical thinking. In sociology we feel comfortable to accept in general terms the antagonistic notions of a workers and a capitalist class within the capitalist mode of

[52] Engels limited knowledge in math is clear from bibliographies presented in the MEGA2 volumes of DoN and AD

production. Even if the form of the workers class is not sharp and blurred with notions as precarious and other forms of subaltern labour, and the discussion on the role of managers as executors of owners of the means of production, which include pension funds also give food for thought. In trying to construct a model with enables the definition of regularities, and hence, calculations and forecasting, we witness that, at present, the royal road to knowledge is a reduction of the multifarious phenomena onto basically linear arithmetic schemes. Even the mathematics of the most advanced idea of non-linearity in General Relativity, where mass and space are mutually defining each other, is casted in rigorous differential geometry, that is to say set theory, which means countable objects as basis. And it works! So what?, the pragmatist will say with good reason. Nevertheless philosophically it remains an open question if human knowledge is stuck to this way of thinking. Can we think ‘out of the box’ to use a stupid slogan, grounded in Euclidean geometry? Or is our thinking so ingrained over the centuries that a reset becomes impossible? Engels adage that it is all numbers and forms might be historically the root of modern mathematics, the many real life phenomena that still can only be “handled” by algorithmic approximations exclude thinking about “another world is possible”. In an entertaining and serious book the former Chess world champion Garry Kasparov who lost infamously in the rematch with IBM computer Deep blue in 1997, discusses the issue to what extent known knowledge (e.g. all openings in chess) can be stored in memory and made operational and to what extent the human mind is able to avoid this by e.g. trying to lay not the best possible move. He also digs deep in the discussion on brute force calculations and the possibility of machine learning in having rules and experience such as is suggested by the AlphaZero computer of the Deep Mind company that won from the Go champion Lee Sedol. The suggestion is that if phenomena are rule based, ambitiously called Artificial Intelligence, calculators can outperform humans. The point again is what part of humanity is rule based and what part does not fit in linear algebra? Kasparov defends the view that computers are still tools and certainly we can agree on that. In other words can humanity represented by an advanced formal algorithm, such as the calculus helps us to handle variable qualities, and enable us to forecast. Or is this brilliant human invention only one of many possible approximate models to curb the problem of haphazardness in a social historical context? {Kasparov, 2018}

9.2. Marxist Economists

Under Marxist economists there is a current often named Hegelian Marxists who analyse the dialectical structure of Hegel’s method in the application of Marx’s economical writings. This is “in and for itself” certainly an interesting field of research. A recent overview of this school, with an emphasis on mathematics, is given in the PhD thesis of Damsma {Damsma, 2015}. Damsma argues that this current of researchers (Geert Reuten, Patrick Murray, Chris Arthur, Tony Smith, a.o.) take a distance from Engels and their “modern” interpretation harks back to Hegel with his systematic-dialectical methodology. Their aim is “to critically examine whether it is methodologically possible to combine mathematical rigor with a systematic dialectical methodology and, if so, to provide an indication of how mathematics may be instrumental to a systematic dialectician and of how a systematic-dialectical

perspective may help mathematical model builders". {p.1}. In a footnote Damsma reduces his study to Marx's schemes of reproduction, and states modestly that "How the indicated guidelines may be beneficial to model builders generally is beyond the scope of what can be achieved in this work". This ambition means that the search is for an equally rigorous mathematics based on dialectical logic as there exists a rigorous mathematic based on formal propositional and predicate logic. A tall goal, as it suggests the ability to formulate rigorous dialectical laws to start with.
He opposes Hegel's approach to Marx as: "Hegel discusses mathematics this object is thought. His deeply philosophical question in this context is: what is it that enables us to think at all? His answer –one that, under his influence, is perhaps obvious today – is that thought requires language. If this is the case, Hegel reasons, the structure of language can inform us about the structure of the world we think about (less obvious, though relevant for the mere possibility of knowledge). Mathematical and formal thinking has a place in this structure of language, but it cannot be directly applied at more concrete levels (e.g. the level of society) without elaborate qualitative empirical considerations about these fields. So, mathematical models may play a role in the empirical sciences, but not in dialectics" {p.3}. Marx on the other hand starts with an analysis of the capitalist mode of production and hence: "Marx sees this world itself as being both qualitatively and quantitatively constituted. So, quantities are an integral part of capitalism, rather than being externally imposed on it. It is this characteristic of capitalism that enables (mathematical) modelling methodology to be integrated with systematic dialectics all the way through, albeit with regard to the study of capitalism only (that is, amongst the systems that Marx knew of)" {p.4}.

However, Damsma is clearly not versed in the foundations of modern mathematics or physics (on which he quotes some popular works). His overview of Hegel's dialectical foundations of mathematics (chpt.2) is of interest, but it is not confronted with modern mathematics and hence remains a philosophical exercise. So though the comparison analysis of Hegel's "systematic-dialectical" approach versus the structure of Marx's *Capital*, is certainly of deep interest, the aim to reach mathematical modelling is not met.
The old problem that dx/dy is going tendentially to zero which might suggested that we end up with zero divided by zero (0/0), and hence becomes ill-defined, was already solved during Marx lifetime, with the introduction of the notion of the limit, a series with no end. Still, ample references are made in analysing Marx dialectical method in writing capital to Marx's mathematical notes. Damsma's whole chapter 3 *Marx's systematic dialectics and mathematics* repeats the outdated discussion on Marx's struggle with the calculus, but doesn't address the pertinent quest how smooth functions (the basis of the calculus) give rise to sublations. At the end of the day Damsma shows that Marx's relatively simple schemes can be interpreted as adhering to the systematic-dialectical approach. Having said that, the reader gets the strong impression that no more has been proven than a casting of Marx trials on relatively simple transformation schemes into a formal model. This, without guidance for the future.

Carchedi {2011} on the other hand develops a novel approach to the problem in which he tries to explicate the dynamics of social reality. "What is missing ……in all participants of the value debate (both on the value-form side and elsewhere) is Marx's *dialectical* view of social reality, the view of social reality as a temporal flow of contradictory phenomena changing from being determining to being determined and vice versa and continuously merging from a potential state to become realised and going back to a potential state. What is missing is the view of social phenomena as both realised and potential, as both determinant and determined, and as subject to constant movement and change" {Carchedi 2011, p.73}. To that order he as well enters the discussion on Marx's mathematical manuscripts.

Carchedi tries to work out a dialectical logic for social sciences, in contradistinction to formal yes/no logic and its true tables. An important aspect is that by such an approach the whole of mathematical reasoning is under scrutiny, as all modern mathematics is based on formal proof theory in which the tautology is an essential element. He defines his first principle as "That reality has a double dimension, what has become realized and what is only potentially realizable, is something only empirical philosophers and social scientists (prominent among them the neo-Ricardians) deny" {Carchedi, 2008b. p.1}. "First, potentials are not, as in physics, elements of realized reality (particles) that exists in their own form, waiting to be discovered. Potentials are not, as in the Hegelian tradition, empty forms waiting to receive a content the moment they realize themselves. Potentials are not, as in formal logic and inasmuch as they play any role in formal logic, attributes of realized reality" {idem p.2). In his appendix three {Carchedi, 2011, pp.279-290} he summarizes his ideas about Marx's mathematical manuscripts. Carchedi states that "Marx studying differential calculus, was seeking both support and material for further development of his method of social analysis. Seen from this angle, the Manuscripts are vastly more significant for the social scientist than for the mathematician or the historian of mathematics" {p.280}. More pertinent is his claim that "Rather, the point is that even though the Manuscripts do not deal with the relation between dialectics and differential calculus, *Marx's method of differentiation provides key insight into what was Marx's dialectical view of reality*" {Ibid. p.283}. The interesting idea is that Marx's thinking is now framed in the challenge to formulate a theory where: "In society, the negation of the negation accounts for the possibility that due to their contradictory nature, social phenomena supersede themselves through the creation of their own conditions of supersession" (ibid. p.288}. For the present author this is an important observation, and contrary to e.g. Engels attempts to formulate the whole trinity of dialectical relations as overarching notion for all sciences. Something which became a doctrine in Stalinism. It also dovetails with my own opinion that there is no compulsory reason that models that work in the natural sciences should work in other fields. "The real question is what and how formal, including mathematical, methods can be developed to deal in specific fields of the humanities. The example of statistics is the only case in which a powerful method was originally developed outside the natural sciences" {Kircz 2015, p.9}. Carchedi uses Marx's notes as argument in the discussion on dialectics and attempts to remain happy with standard formal logic. In a way this is correct, provided that we accept Marx smelled a rat, but in fact catched a red herring, as the calculus as a science changed dramatically even during his lifetime. What remains is the pertinent issue of to what extent can we apply successful mathematical methods in natural science in social sciences, and why the marching order should be to invent 'mathematical' models that go beyond formal logical systems that work in statics and which only by approximation deal (most successfully) with dialectical dynamical systems.[53] This on top of the ever green question: what is continuity???

9.3. Back in the USSR

The reader might ask: why crying over spilled milk? Indeed the most pragmatic way is to burry all Stalinist nonsense about mathematics and forget it all. However, from a historical standpoint it is important why often bona fide socialists got trapped in an anti-creative ideology and why even today people believe and/or try to defend this ideology. Apart from that, the discussion on what is the materialistic basis of our thinking and are we - in principle - capable to transcend our bodily/ neurological firmware in thinking up to ever novel models in our quest to understand our position as part of nature. It is not a big thing to say that all human thinking is a physical activity. The question is what the mechanism is that we can think-up, most presumably induced by sense-impressions, ideas and models that are or fantastic, or only become valuable as model much later. The standard example again in science is that Albert Einstein was struggling with his mathematics and then his friend the mathematician Marcel Grossmann suggested that the notions of differential geometry and tensors might be of use. Before that time tensor algebra was a typical totally abstract field void of any suggestion for an application. And, indeed, after Einstein's publications on general relativity this mathematical field blossomed. Hence, the quest is: if and how novel ideas come to the fore in an ideological environment hostile to classical western thinking, e.g. in a socialist society?[54] This issue is particular important because in the historical materialist tradition it is key that the emergence of new science or at least the success of new science is dependent of the socio-economic environment. This can mean various things. It can mean that a new idea can only become a serious quest if the environment is ready for implementing it. But it can also mean that an environment induces -more or less compulsory (like new armoury in the industrialisation of war) - the searching for finding new avenues. Here the standard example is the heliocentric world view, which was already very old before Copernicus did his calculations and it took quite long before the idea was accepted as superior to the geocentric view, which for all practical earthly purposes is still doing quite well. The sun still comes up in the morning and goes down in the evening. But as you never know where you find some new idea and as Louis Pasteur famously said: “mais souvenez-vous que dans les champs de l'observation le hasard ne favorise que les esprits préparés”, we

[53] It is important to note that also in modern modal logical approaches time (or flow) is still a hard nut to crack. Tense Logic starts with a propositional system with operators such as: 1) at least once in the future, something will be the case, and 2) at least once in the past, something has been the case {Van Benthem, 2010, p.207}. However, in many cases we simply cannot even start with these propositions, because we deal with potential possibilities and then firm standard logical operations are not enough.

[54] Note that {Kojevnikov, 1999, 2002} emphasised how novel directions, based on the idea of collective motion, in physics were influenced by communist thinking.

have to go on. A secondary reason is that very frequently people want to forget the unsympathetic role of an important person, because it is only the result that counts, independently of the collateral damage. Take for instance the introduction to the decimal system by Napoleon in continental Europe. [55]
Above, we posed the question: how to deal with maths if both Marx and Engels based their mathematical ideas on obsolete knowledge? Can we accept it because the underlying reasoning is still sound and the examples are just plainly wrong? This is not the same as in experimental sciences. When we realised that the chemical atom was a composite, atomic theory changed for good. But, if we inductively design a theory (or parts thereof) based on other theories (e.g. the proof of the fundamental notion of the negation of the negation by mathematical gymnastics) then the issue is less simple. Because the original idea of a negation, that was ever a negation of something else (or in Hegelian terms a positive negation) might be still a stepping stone towards new understanding.
In the Stalinist cult, people had to bend over backward to proof their pledge of allegiance to the flag of the Central Committee. For that reason in scientific works we see standard formulations about the genius of Marx and Engels sprinkled thorough the text without explication what the concrete relation is with the main message of the text.

As Leon Smolinski (1973) points out looking for novel ideas on mathematics by Marx we have to take his goal (qualitative) ruptures in economy into account, whilst Marx struggled trying to get grips of mathematical economics. Leaving aside Marx’s calculations in *Capital Vol.III*, which are more examples than mathematical analyses, Marx was fascinated by the calculus in the hope it could help to describe the motion or development of economic processes. However, the calculus is a system that deals with the development of an endless series of very small steps. It demands, as said above, smooth functions. Smolinski {1973, p.1199} states: “But in Marx's highly polarized system, economic progress takes place through violent, discontinuous change. For him, nature does engage in revolutionary leaps which are of crucial importance. The functions involved do not have derivatives at these stages on which Marx focuses his attention. Calculus is applicable to the phenomena of the type "a little more" or "a little less" studied by marginalist economists but not to the "either/or" phenomena which form the cornerstone of the Marxian economic system. For him the key fact is that a commodity has value or does not have it, labor is productive or is not, a participant in the economic process is a capitalist or a proletarian, society is capitalist or socialist. For this polarized universe a binary calculus might be a more suitable tool than differential calculus, and it was probably no accident that toward the end of his life Marx seems to have become interested in finite mathematics”. During the bureaucratic counter revolution, “Mathematical economists were accused of concealing anti-Marxist concepts "behind ... the impenetrable logarithmic wall,"[56]
Subsequently, mathematical economics was proclaimed to be "the most reactionary brand of bourgeois economics" (Kol'man 1930). It was at that time that the myth was adopted of the inapplicability of mathematical methods in Marxist economics and of Marx's alleged

[55] A **Error! Main Document Only.**standard in the history of measures is the elaborate study of the Polish Economical Historian, Witold Kula: *Measures and Men, {*Kula, 1986}

[56] This is a reference to Marx’s struggle with geometric progressions.

opposition to their use". This resulted in the original disinterest in the works of e.g. Kantorovich (see also below).
The outstanding question is in fact not what Marx hoped for, in his mathematical studies, but what kind of mathematical methods are needed and must be developed that are able to describe ruptures and breakdowns. A question which keeps many people buzzy after the financial crisis of 2008, which was prone to the confidence that so-called technical analyses of stock prices reflect an economic (and mathematical) reality.
Therefore is it interesting also to discuss two world class mathematicians who remained loyal to the regime. One is the Dutch mathematical hero and MIT professor Dirk Jan Struik and the second is Academician Aleksandr Danilovich Aleksandrov, rector of the Leningrad University (1952-1964) and later star of the Siberian branch of the academy of sciences, proud recipient among others of the Lenin order, the Labour red banner order (4x) and Winner of Stalin Premium of the II degree (1942) and so on. So, serious people.

But first we will dig a bit deeper in the history of the people who are often quoted in relation to Marxist Mathematics, and therewith became common references in the discussion. Hence, became an authority by statistics.

9.3.1. Sofya. Yanovskaya

Sofya Aleksandrovna Yanovskaya (1896-966)

Yanovskaya in her preface of the 1968 Russian edition {Marx, 1983} gives an overview of Marx increasing interest in mathematics as become clear from his correspondence with Engels, e.g.: in the May 31, 1873, Marx writes to Engels: "I have variously attempted to analyse crises by calculating these 'ups and downs' as irregular curves and I believed (and still believe it would be possible if the material were sufficiently studied) that I might be able to determine mathematically the principal laws governing crises" {MECW V44, p. 504}. This more or less is the first expression of hope that the immanent crises in the capitalist mode of production might be fit into a mathematical expression. A hope, keeping Marxist economists busy until today.

Obviously Yanovskaya refers, as is the custom, a good deal to Engels. " With the introduction of variable magnitudes and the extension of their variability to the infinitely small and infinitely large, mathematics, usually so strictly ethical, fell from grace; it ate of the tree of knowledge, which opened up to it a career of most colossal achievements, but at the same time a path of error. The virgin state of absolute validity and irrefutable proof of everything mathematical was gone for ever; the realm of controversy was inaugurated, and we have reached the point where most people differentiate and integrate not because they understand what they are doing but from pure faith, because up to now it has always come out right." {Engels 1878, AD p.81} and continues with "Naturally Marx was not reconciled to this. To use his own words, "we may say that 'here, as everywhere' it was important for him 'to tear off the veil of mystery in science' {Marx 1983, p. l09). This was of the more importance, since the procedure of going from, elementary mathematics to the mathematics

of a variable quantity must be of an essentially dialectical character (namely the step form a fixed position to a flow-jk), and Marx and Engels considered themselves obliged to show how to reconcile the materialist dialectic not only with the social sciences, but also with the natural sciences and mathematics. The examination by dialectical means of mathematics of variable quantities may be accomplished only by fully investigating that which constitutes 'a veil surrounded already in our time by quantities, which are used for calculating the infinitely small- the differentials and infinitely small quantities of various orders'. (*Marx-Engels*, *Werken*, Vol.20, Berlin, 1962, p.30) [57]. Marx placed before himself exactly this problem, the elucidation of the dialectic of symbolic calculation, operating on values of the differential." (p.xi}
The question is after all: did Marx succeed in solving this problem and/or did his acolytes were able to do so? Marx tries to solve it by posing the issue as that the quantity dx/dy transforms into *operational symbols*. Yanovskaya, who will end her carrier as an academic mathematician squares the circle by saying: "Marx was not acquainted with contemporary rigorous definitional concepts of real number, limit and continuity. But he obviously would not have been satisfied with the definitions, even if he had known them. The fact is Marx uses the 'real' method of the search for the derivative function that is the algorithm, first, to answer the question whether there exists a derivative for a given function, and second, to find it, if it exists. As is well known, the concept of limit is not an algorithmic concept, and therefore such problems are only solvable for certain classes of functions. One class of functions, the class of algebraic functions, that is, functions composed of variables raised to any power, is represented by Marx as the object of algebraic' differentiation. In fact, Marx only deals with this sort of function. Nowadays the class of functions for which it is possible to answer both questions posed above has been significantly broadened, and operations may be performed on all those which satisfy the contemporary standards of rigour and precision. From the Marxian point of view, then, it is essential that transformations of limits were regarded in the light of their effective operation, or in other words, that mathematical analysis has been built on the basis of the theory of algorithms, which we have described here." {page XV}
In other words although Marx was not on top of the subject and even would not like, it if he knew, he was right.

---

[57] Though often just re-quoted, it seems to be misprint. In the German DoN 1962 Dietz edition the quote is on page 530. "Das Mysterium, das die bei der Infinitesimalrechnung angewandten Größen - die Differentiale und Unendlichen verschiedener Grade - noch heute umgibt, ist der beste Beweis dafür, daß man sich noch immer einbildet, man habe es hier mit reinen „freien Schöpfungen und Imaginationen"(this a jab to Duhring-jk) des Menschengeistes zu tun, wofür die objektive Welt kein Entsprechendes biete. Und doch ist das Gegenteil der Fall. Für alle diese imaginären Größen bietet die Natur die Vorbilder". In the English version MEW V25 p.545. it reads: "The mystery which even today surrounds the magnitudes employed in the infinitesimal calculus, the differentials and infinites of various degrees, is the best proof that it is still imagined that what are dealt with here are pure "free creations and imaginations" of the human mind, to which there is nothing corresponding in the objective world. Yet the contrary is the case. Nature offers prototypes for all these imaginary magnitudes".

It is unclear to what extent this operationalist escape fits in the idea of finite quantities. Yanovskaya then goes on with noting Engels' statement that the turning point in mathematics was the introduction of *variable quantities,* and Engels' discussion on the dialectics of variable quantities in his *Dialectics of Nature.* The key 'mystical' problem is that the actual infinitely large or small remains are "not introduced by means of operations of mathematical grounded consistency but are hypothesised on the basis of metaphysical 'explanations'... The practical use of the calculus then becomes for a collection of tricks." It is obvious that Marx struggled with the limits of mathematics in his time and tries to ground this in the idea of countable objective small entities. This idea came to a grinding halt at the end of the 19th century and the whole calculus was rephrased in the language of set theory (see above).
Yanovskaya's scholarly preface is also a good review of Marx' struggles to overcome the foundational problems with the calculus. However, it is a bit strange that the idea of promoting the differential dx/dy to the level of an operation, a kind of practical shortcut of a longer algebraic approach, is presented as a great novelty. Marx, who adopted an algebraic approach and felt save by the approach of Lagrange to expand the differential expression in a (Taylor) series, so that higher terms could be discarded, keeps the tension between fixed and variable values. Obvious the fundamental problem remains: what is motion and how does a fixed value starts being a variable value? This problem is not solved by e.g. declaring that fixed states do not exist in reality and everything is continuous flux.

In the same publication {Marx, 1983, pp. 235-255}, an article by Kol'man and Yanovskaya (although here her first name is now spelled Sonia) on Hegel is republished {Kolman & Yanovskaya (1931)}. This paper is a typical statement in the lingo of those days, starting with the apodictic: "The enormous interest shown in the study of Hegel by science in the Soviet Union is best justified in Lenin's philosophical legacy" etc. In itself this is clear exemplar of the fact that in that period the philosophical debate between the so-called Deborinites (dialecticians) and the so-called Mechanistic fraction, mainly natural scientists who based themselves more on positive science than on philosophy, was getting out of hand. Ultimately the last current was in an authoritarian way demolished by state and party intervention. Engels' *Dialectics of Nature,* which was published in 1925, became a weapon in the struggle as "Marx and Engels" declared also here that dialectical materialism was a universally applicable model for everything. It brings us to the core of our chpt X, what does one mean with Dialectical Materialism and what is the relation with the state religion Diamat [sneak preview: Nothing].[58]
Interestingly, here, as well as in many other contributions, we witness strong partisan declarations and attacks, mostly based on authoritative quotations, without reference to content. [59]

[58] For this in principle highly interesting debate, which unfortunately drowned in vicious fraction struggles in the building of Stalinism as a state religion, see in particular {Joravsky, 1961} and a participant's account by {Yakhot 2012}. In my opinion we have to pick-up this debate again, but stripped from the political infighting of that time, see also chpt X].

[59] A good example is the almost hatred towards the intuitionist school of Brouwer, which played an important role in the development of constructionist mathematics, and hence

First we will deal further with the sinister figure of Kolman, and then we turn, as said, to the exquisite Dutch-American communist mathematician Struik and will end with A D. Aleksandrov, not be mixed up with the theoretical physicist P.A. Aleksandrov (1896-1982).

9.3.2. Ernst Kolman

One of the most remarkable and sinister people in the field of DIAMAT and mathematics is Ernst (or Arnoŝt) Kolman (1892-1979). Kolman was born to a Jewish mother and a Czech father in Prague. He studied at the 'Prager Tschechische Technische Hochscule', as well as that he followed classes at the mathematical division of the philosophy faculty at the Charles University. As soldier in WW1, he became a prisoner of war of the Russians and was liberated by the revolution. He joined the communist party in 1917. Clearly Kolman was a bright man, endowed with strong ambitions. Within the party he was a typical climber and survivor who played an important role as ideologue and partisan in the Stalinist purges. His greatest feat was the purge of the Moscow Mathematical Society {Seneta, 2004}, which was the centre of top-notch work, but carried out by Russiam-Orthodox mathematicians. This so-called "Luzin-affair", also mentioned above , named after one of the most important victims (but he survived) Nikolai, N. Luzin (1883-1950) is well described in the literature {e.g.: Graham 1987, Graham &Kantor 2009, Lorentz 2002, Senica 2004, Gordin 2017, Demidov 2016}. Also in the Ukrainian purges of 1933, Kolman played an important role {Ford 1997}.

The life story of Kolman is certainly interesting. After WW2 he returns to Prague and became chef of the propaganda department of the central committee of the Czechoslovakian communist party, and a year later professor in philosophy in particular on Dialectical- and Historical-materialism, Logic, and Ethics. After critique on party chef Rudolf Slánsky (1901-1952), he was send back to Moscow in 1948, where he spends 3 years in jail. He was released in 1952.[60]

In 1959 he was back in Prague as director of the Philosophical Institute of the Academy of Science. From 1962 till 1976 he was again in Moscow. That year he writes an open letter to Leonid Brezhnev in which he, 84 years old and after 58 years party membership, criticizes

---

programming, and claims that Marx was the inventor of algorithmic thinking. It is understandable that not many people like to dig into this mud in a faint hope to rescue materialist philosophy.

[60] Slánský was found guilty of "Trotskyite-Titoist-Zionist activities in the service of American imperialism" and public hanged at Pankrác Prison on 3 December 1952. His body was cremated and the ashes were scattered on an icy road outside of Prague, as the car carrying the urn was slipping. In Kolman's interview with Janouch (Kolman 1982, p. 330)] he says: "Es ist ein Ironie der Geschichte - oder bescheidener: meines Lebens -, dass ich im September 1948 in Prag gerade deswegen verhaftet und in die Lubjanka nach Moskau gebracht wurde, weil ich Slánský beschuldigt hatte eine Politik der Distanzierung von der Sowjetunion unter der KPdSU zu betreiben (diese Behauptung war nicht unbegründete". Is this ironic or another example of Kolman's love for the dialectical law of the negation of the negation?

the regime and defects to Sweden, where he dies three years later {Kolman 1982, Gordin 2017}.
Kolman’s wanderings are interesting not only because his enormous output in papers and books, but also because he tries to come to terms with his miserable past in his autobiography under the modest title *The mistaken generation; This is not the way we should have lived* . {*Die veriirete Generation; so hätten wir nicht leben sollen, Eine Autobiographie.* Including: *Wie habt ihr so leben können? Ein Dialog zwischen František Janouch und Arnošt KolmanI*, Kolman 1982}.[61]
The book is a mixture of self-centred self-analysis and criticism, as well as a defence of his version of Marxism, and boosting his achievements. Certainly, due to the publishing of his open letter to Brezhnev in which he renounce his party membership, in *The New York Times* on 13 October 1976, he got some credibility as Czech oppositionist. The fact that in his memoirs about 100 pages are complemented with an interview by his son in law František Janouch, a noted nuclear physicist, former member of the Communist Party and member of Charta 77, under the title: : *Wie habt ihr so leben kőnne,* adds to his prestige in certain circles. But as Ford (1997, p.355) says: “Here [about his stay as party executor in the Ukraine-jk] as elsewhere in his memoirs, Kolman reflected on the destructive role he played in relation to other party members. He did not, here or elsewhere, reflect in a comparable way on this role in the destruction of the peasants or others who were not communists”. The difference with this book and the famous 1949 collection *The god that Failed, a confession* {Koestler, 1949} is that Kolman remains convinced of his philosophical outlook. This makes his works interesting. Gordin (2017) writes that “... when communism was dismantled across Eastern Europe and the Soviet Union, dialectical materialism essentially died with it” , and “ this was one of the twentieth century’s most vigorous philosophies of science...” . I’m afraid; this enthusiasm of Gordin is more based on the propagandistic power of its priests than on solid understanding of DIAMAT, not to say Marxism.
Most of the discussions on DIAMAT and its players are to a large extent, often fascinating, historical overviews of ego and power fights, but hardly deal with the content of a (perceived) dialectical world-view. The discussion was frozen in the early ‘30s in the USSR, after the debates between the dialecticians and the mechanists, as I remarked above. After those years, DIAMAT became a *pars pro toto* for all philosophy truths embodied in the policies of the Central Committee of the Communist Party. Because in the present paper the aim is to go back to content, we will try now to unearth some content from the writings of Kolman.
Kolman wrote a very large number of papers of which part is listed in his memoirs {Kolman, 1982}, not many are available in English, and hence the selection discussed below might be a bit lopsided. As mentioned above, he presented 3 papers in the collection of papers in the 1931 conference in London {Kniga 1931}. The first paper, already mentioned is the short list of Manuscripts of Marx on science and technology. The second paper has the title “Dynamical and statistical regularities in physics and biology”. In the style of that time it begins with an attack on idealist bourgeois, Machian, thinking and the flight from materialism. Then claims: “ In the Soviet Union, where the foundations of Socialist economy are nearing completion (the first so-called five-year plan started in 1928-

[61] Seneta (2004, p.363) compares the various editions of the book.

jk), where, consequently, the basis for free play for market forces is continually growing narrower, being thrust aside and replaced by socialist planning, questions related to the validity of statistical methods, to the reliability of predictions, to methods of planning as a whole, are extremely acute and cannot be answered in the absence of a sure methodological basis" (p.2). It goes without saying that the last part of the sentence is trivial. It then goes on in a wide ranging expose touching many fields to "prove" that mechanistic, that is to say without taking concrete societal factors into account, mixes up the difference between the general and the particular, in biology as well as in physics. Concretely: "Statistical laws lose their scientific value, if the essential aspect of the phenomenon is forgotten, if the particular is metaphysically denied of being dialectically handled" (p11). Hence, materialistic dialectics is in demand.
This subject will become one of Kolman's enduring interests. The background, of course, is the belief that proper planning gives pertinent results. The severe ideological intrusions, without proper mathematical knowledge of fields, like probability theory, in development played a devastating role not only to the personal carriers of even lives of its practitioners but without doubt on the development of a planned economy.[62]

Quoting Seneta: "What is important is Kolman's (...) in the demise of Soviet statistics and statisticians in the name of Marxism and dialectical materialism. These attacks, to be effective, could only be constructed as attacks on political unacceptable *statistical analysis* of data, and not on *statistical theory*."
In his semi-religious overview Kolman is quoting Lenin: "–practice stands higher than (theoretical) knowledge, for it possesses the distinction not only of general validity, but also of direct reality". Though Lenin only warns against adding up unequal quantities like poor and rich farmers. It becomes nearly funny when Kolman says: "Stalin in a speech delivered in April, 1929, developed this idea by applying it to conditions in the Soviet Union at that time, conditions which have already been completely changed" (p11). The need for proper techniques for measuring and forecasting is clear from this sentence. Or in his own words: "... we must study concretely this negation, of negation". (p12)
Sick humour aside, the real issue is indeed the development of theories of forecasting based on past performances and including the (incremental) changes in the very process. Pure simple statistics is always a momentary picture. Dynamical forecasting (if you want so, the negation of the pure probabilistic approach) can make use of statistics but is presently still a dream of many a factory or stock market manager, not to think of an economist. It works in quite times, cf. the financial crisis of 2008 and the present Corona crisis.

The third paper has the promising title "The present crisis in the mathematical sciences & general outline for their reconstruction". It is not uncommon to talk about a crisis in a field if various schools collide head-on in approach, and no hegemonic approach to a certain problem is reached. In that sense active vibrant science is always in crisis and this is the way it should be. Unfortunately this up-beat paper is more an ideological treatise than an effort to give insights in what the answer is to the "present crisis in the mathematical sciences ...

[62] Seneta (2004), as well as Siegmund-Schultze (2004), enter into the details on the issue of probability theory, both based on the original Russian sources.

(taking) into consideration the crisis in bourgeois science as a whole...". Quoting Lenin (1908), who at the time of writing could not have any solid understanding of the new physics of the beginning of the 20th century, is part of the Stalinist cult. Lenin's alarm is about the discovery of radioactivity which was suggested to allow matter to disappear and the mechanistic interpretation that philosophical materialism therewith is becoming obsolete. Lenin's point is philosophical materialism.[63]

Also in his list of pure mathematics crises (the infinity discussion, probability theory, the logic and intuitionistic school on foundations of mathematics, etc.) the tenor is the simplistic and 18th century view that a materialistic word view is dealing with solid objects only, as well as that all mathematics must have a direct tangible object. Instead of taking the problems with the new physics and the mathematical break troughs as challenges to expand our 'materialist' worldview, a 'believe system' is arrested. Typically for the ossified DIAMAT religion, Kolman's paper, like many others is a painful attempt to on the one hand accept the progress in the field and on the other hand to declare its intrinsic features dangerous and bourgeois. No way, have we seen a glimmer of hope, how the new sciences might transcend the crisis in a higher - dialectical materialist - human understanding of the world around us. The Tibetan prayer wheel, telling us that we have to tackle the issues dialectically goes merrily round and round, without becoming explicit what that means other than repeating that all knowledge is historical contingent and we seem to have three - trans-historical - eternal dialectical laws. But having said that: there is more at stake, as this approach favours certain political developments.

The manifest problems with a plan economy are one of the Achilles' heels of the construction of a socialist state. Right from the start of the USSR, many bright people tried to tackle this problem. The idea of a rational economic planning as an answer to the free market war of everybody against everybody in the capitalistic mode of production, fitted in the technological utopian ideas after the revolution, and became, in an ossified form, the backbone of the notorious five year plans.[64]

As known, the complete bureaucratised and centralised 'plan-economy' of the USSR came to a grinding halt in the final days of Stalin's regime. The rational socialist planning needed more than slogans.

The first "system-builder" was A. Bogdanov (Alexander A. Malinovsky 1873-1928). His clearly "modern" world-view which he called empiriomonism was based on a positive "scientific" attempt to surpass traditional Marxism and develop a new all-embracing theory of the 'Living Experience', which he named Tektology.[65]

---

[63] E.g. " Yesterday's limit to our knowledge of the infinitesimal particles of matter has disappeared, hence—concludes the idealist philosopher—matter has disappeared (but thought remains). Every physicist and every engineer knows that electricity is (material) motion, but nobody knows clearly what is moving, hence—concludes the idealist philosopher—we can dupe the philosophically uneducated with the seductively "economical" proposition: let us conceive motion without matter.... {Lenin, 1908, p283}.

[64] The names of Preobrazhensky and Bukharin, are important in this debate.

[65] For an insightful paper on Bogdanov's broad interest in science see Gare (2000). For a full discussion of his *Philosophy of Living* experience see Jensen (1978). For Bogdanov's mature philosophy see: Bogdanov (2016). For a comprehensive overview of systems thinking in the USSR,

At present, friend and foe recognise that Bogdanov's Tektology is the first attempt to develop an encompassing (technology and sociology) system theory or cybernetics {Susiluoto, 1982}. As Bogdanov was anathema during the Stalinist era, his work was completely "forgotten" and no references to these first inroads to system theory are made by the Diamaters.
Operational research, process engineering, or cybernetics are fields that try to compute process and are able to "learn" from results. The notion of "feedback mechanisms" is crucial here. The field got a kick start with Norbert Wiener {Wiener 1948}, and served the pipe dream that the emerging computers ultimately could regulate production and society, and in its sci-fi variant even humankind. The rocky road of the field in the USSR is also well described by Graham {Graham, 1987, chapter 8: Cybernetics and Computers}. The hope that cybernetics with its mix of selectivity of information and reactive decentralisation of control (like the human body) could safeguard the economy became a central theme.
The use of mechanical calculation became a necessity in the military during the Cold War. In particular the ideological issue of to what extent mechanical computing could be used as a metaphor for social and biological research created a hostile and secretive atmosphere in the development of a native computer industry. Only in Khrushchev's thaw the necessary openness became possible.[66]
In 1958 the Academy of Sciences created a Scientific Council on Cybernetics and in 1961 the Communist party endorsed it as one of the major tools for the creation of a communist society {programma kommunisticheskoi partii Sovestskogo Soiuza, Moskow 1961, as quotes in Graham 1987, p. 271}. Interestingly, it was the ideologue Kolman who already in 1954, understood the changing winds, and gave a lecture to the Academy of Social Sciences of the Central Committee of the communist party in which he attacked the then dogmatic 'materialist' critique on cybernetics. His speech became a starting point for a deep discussion on the issue. The success was fuelled by the work of important mathematicians on control systems and work on thermodynamics, as cybernetics is seen as a way to reduce chaos, or in technical terms the increase of entropy. Kolman's speech was followed up by a paper in 1955. Obviously, Bogdanov's work was still completely ignored, most possibly because it was not any more publicly available, but Kolman (not the only one in the debate) was old enough to remember.
For Diamat this reformulating of processes in terms of information and order became a discussion issue of how to square this with their understanding of Marxism. "In a 1964 article prefaced with the slogan 'Only an Automation? No, a Thinking Creature' Academician [and internationally renowned mathematician, and once student of Brouwer-jk] Kolmogorov commented, "The exact definition of such concepts as will, thinking, emotion, still have not been formulated. But on a natural-scientific level of strictness such a definition is possible...The fundamental possibility on creating living creatures in the full sense of information and for control, does not contradict the principles of dialectical materialism" {Graham 1987, p. 279}.

---

comparing Bukharin and Bogdanov and beyond see: Susiluoto (1982).

[66] In a fascinating study {Gerovitch ,2001} discusses all elements of this discourse - secrecy, ideology, bureaucratic infighting, etc. in comparison with the mirror discussions in the USA.

This is not the place to advance further on the issue of what information is, also in a sociological sense. However, it is important to note that in Kolman's thinking the fundamental notion that we deal with countable entities, the same way Engels says in his definition of mathematics as the field of forms and numbers is consistent. It is a point of further discussion to what extent this atomistic approach relates to the 'antagonism' of discrete vs continuous. Susiluoto {1982, p. 168} (discussing Graham's {1967, p.93} more positive and Joravsky's {1970, p.361} most negative assessment of Kolman) states:
"... the contradiction in these assessments disappear when Kolman's ideas are explained in the light of Stalin's linguistics. According to Stalin, language could be placed in a separate category, relatively independent of the dialectics of base and superstructure. Thus linguistic research also constituted a relatively independent area of its own. According to Kolman, formal logic and cybernetics were a "powerful instrument of cognition within their own framework". Thus it was possible to focus the attention of "linguistics" on the language of cybernetics. Stalin had seen grammar as a set of abstract relations comparable with geometry. Kolman, for his part, put cybernetics on a par with mathematics. In this case, it was easy to show that the interpretation which emphasized the abstract aspect of cybernetics was in agreement with Stalin's line. According to Stalin, language had one thing in common with e.g., machines in that it served both capitalism and socialism. What justified the existence of linguistics was that it "organised people for joint work". [67]
Kolman showed how cybernetics severed capitalism. And it was natural to ask, especially since cybernetics has been "purged" of its ideological elements, why it would not serve socialism as well". Susiluoto's study also discusses the earlier debates on Taylorism in the twenties.[68] Obviously, it all ties in with the central discussion, also in this study, to what extent (consistent) methods once developed in a certain socio-historical context, keep their validity and usefulness in another context? It is a central tenet of the de-ideologization strategy of scientists (see also {Gerovitch, 2001}).

Kovaly {Kolvaly,1972} discusses Kolman's 1962 paper "views into future" where Kolman develops the notion of the Mentalcaptor "that will change the world into a paradise on Earth". This contraption is supposed, using cybernetic techniques that we put on our heads and will enforce our major thought processes. It will also be able to exchange signals with other brains. Kolman must have read Bogdanov's SciFi books!
The same year he writes: "Our ideal is the happiness of man. Happiness to work for the Communist Party. Our goal is to all be happy that we have done something for the

[67] For Stalin's peculiar opinions on linguistics, in relation to the theories of Nicholas Y. Marr (1895-1934). See: J.V. Stalin (1950). And on the discussion see o.a. Kojevnikov (2000), and Boer in his Stalin biography (2017, p.57) who has his own more friendly interpretation of sentences like "'Culture may be bourgeois or socialist, but language, as a means of intercourse, is always a language common to the whole people and can serve both bourgeois and socialist culture." Intrinsically avoiding the class-laden aspects as researched in Pragmatics (the languages as it is spoken).

[68] Taylorism was seen by many as a possible answer to the chaotic organization of the post-revolution factories. See {Bailes, 1977}

Communist Party" {quoted in Kovaly, 1972}. Obviously the Cold War SciFi phantasies on brainwashing found a keen defender in Kolman.

On this point it is important to mention the big difference between fundamental and applied mathematics. I think it is related to the issue of the different spheres of production and circulation in the economy, more concretely with the difference between value and its representation price in the market. In the Diamat's economic discussion labour value was obviously the central starting point. But how to dovetail this notion with the practicalities of a centralised plan economy? Only in the Khrushchev thaw, existing mathematical modelling became generally accepted research topics and seen as ways to overcome the grid-lock situation. The most important example is the work of the mathematician Leonid Vitaliyevich Kantorovich (1912-1986) who already in 1939, based on practical work in the plywood industry, introduced optimising methods for multi-variable problems, later named Linear Programming. Generalised, his methods became a theory for the allocation of all scarce resources, and hence ways of maximizing the effectiveness of investments (expenditures) in wages, opportunity costs, etc. Kantorovich 1939 seminal paper was totally ignored at the time of writing, but got prominence in the early '60s. Earlier the field of 'mathematical methods in economics' already flew high in the west and Kantorovich methods were reinvented as the Simplex method in the US. In an enthusiastic contemporary review of the works of Kantorovich and the economist V. V. Novozhilov,{Campbell,1961} colludes that finally with this development the Labour theory of value was discarded and 'real' prices are back in Soviet economics. Apart from the problems that a) the Diamat economists were not examples of creative 'Marxist' economists and b) the author don't understand the whole notion of the difference between value and price, the episode is an excellent example of the tensions between good working applied mathematical methods in a certain period and its dynamics, in this case the further collapse of the USSR centralistic economy, and the lack of proper theories for non-market based economies.

Equating the USSR with communism and then declaring the idiocy of communism, does not touch on the essential question of how applied mathematical methods can be useful in other economies, as all types of economy will have to deal with the optimisation of scarce resources.[69]

Happily for Kantorovitch, not only was he later on recipient of the Stalin and Lenin prize but shared, with the Dutch born Tjalling Koopmans, the Swedish National Bank's Prize in Economic Sciences in Memory of Alfred Nobel, in 1975, the same year Andrei Sakharov got the Nobel peace prize, but was not allowed to collect it.

The issue of the discrete versus the continuum is mentioned by Kolman in a not very clear paper on a unified theory of matter [Kolman 1935] in which he deals with the idea of the

---

[69] Smolinski, {Smolinski, 1964} gives a nice overview of the problems with the crisis in Soviet central planning and the new call for mathematical tools. This dovetails with the sudden popularity in cybernetics.

then new discussion on a minimum value of time intervals. Kolman proposes: “...to proceed from the discontinuity of time and the continuity of space...”. This hybrid approach demand, according to him, an “adjustment” of tensor analysis, not a clear proof of his understanding of that branch of mathematics. However the reader has to understand that: “It is necessary to emphasize with all clearness that the discontinuity of time is by no means conceived by us in the sense of absolute discontinuity, the “atoms” of time, or chrons, are not presumed to be final indivisibles”. In a footnote he suggests that such a result “may be splendidly made use of by idealists and theologians”. Not a very convincing argument as every scientific result can be made use of by theologians, as Diamat itself is proving.
In his latter days, he desperately tries to reap international recognition as Gordin (2017) illustrates from the correspondence with his old friend Dirk Struik (see below). Struik and his fellow fellow-traveller R.S. Cohen (a famous philosopher of science from Boston Univ. and a productive editor of books) arrange an “occasional” paper of the American Institute for Marxist Studies (AIMS), which Cohen and Struik introduce as well as edit. They praise their friend Kolman for introducing: “cybernetics to Marxist thought in East European countries” and mention his participation in the 1931 conference as well as above mentioned paper from 1935, without comments. In some way we may consider this paper {Kolman 1965} as his concluding “considerations about the certainty of knowledge” in his long career.
Starting with that: “the foundation on which mankind built and continuous to build the structure of knowledge of the world are matter, space and time”, and continues that this knowledge is accrued by physical experiences. He then argues that the “assumption concerning the constancy of special dimensions of a body, independent of its position in space, can by experiment, *neither be proved nor disproved”*. Here he refers to pre-Einsteinian physics, where we deal with the invariance of an object in 4D space-time, independent of the coordinate system. He goes on with the notion that some notions are invariant, that to say remain constant in general terms. Strange is the paragraph which says: “Let us remark that the assumption that the amount of matter and its motion is not invariant, (1) does not contradict the well-known physical laws of conservation, because the possibility of change is supposed to be unmeasurable in principle, (2) does not -despite the view widely accepted by Marxists - philosophical materialism, because the *unique* “Property” of matter, whose recognition is *condition sine qua non* for materialism, is to be objective reality, existing independent of our consciousness, and (3) has nothing to do with religious ideas about the creation and the doom of the world since the assumed hypothetical changes may occur as the result of some far unknown natural regularities than happen in a supernatural way, arbitrarily by the ‘will of God’”. And then the text goes on with the discussion on how to derive knowledge from this hotchpotch of ill-defined notions. Kolman then declares “I want to stress that here we are not concerned with the fact that our knowledge is incomplete and inexact, and partly of a relative character, gradually improved by evolution, In this sense, the claim that knowledge come asymptotically nearer to absolute truth is upheld”. A very strange conclusion for a historical materialist who is supposed to defend that knowledge is always a result of socio-historical situations and hence never will find an endpoint, suggesting an endpoint is pure idealism and even worse than Kant. It is tempting to quote Engels on Dühring (Engels 1878- AD p.6) “I could make up my mind to

neglect other work and get my teeth into this sour apple. It was the kind of apple that, once bitten into, had to be completely devoured; and it was not only very sour, but also very large." Fortunately "our friend" Kolman has no influence anymore and this paper is only 12 pages, but this swansong nicely published by two important communist university professors, does not learn us anything about dialectical logic and leads to; "The main goal, therefore, of these considerations is a *purge from native ideas*, which for different reasons stole their way into Marxist philosophy. There is no place for them in a scientific philosophy. One of these naive ideas is the belief that constancy of special, temporal and material measures can be experimentally demonstrated; others are the idea that logical processes have absolute validity, that the mentality of man is identical; and other, such as the possibility to know everything, and the consideration of rising evolution as the universal regularity of all beings". But after this modest statement he posits that: "Truth is indivisible, and must be the same for all. Those of opposite opinion resemble the priests of ancient Egypt". Amen.

This is a very strange conclusion as it not only claims eternal truth, but also violates all experiences and theory laden ideologies, such as those of the Diamat priests. He ends heroically with attacking the idea that ideologues of imperialism see the new introduction of cybernetics as a withdrawal from Marxism. "... we must improve and deepen our ideological contest, challenging the arguments of idealism, mechanism, and agnosticism by detailed, specialized knowledge, never being on the defensive but always on the offensive, along the whole front". (p.12)

With this last pathetic battle cry we still have learned nothing, throughout his long criminal live, about dialectical logic and/or mathematics.
An interesting overview of Kolman's instrumental philosophy is given by the Czechoslovakian scholar Pavel Kovály (1928-2006) {Kovály 1972}, citing many publications. His verdict on the man who lost his religious believes in Stalinism only after he became a victim himself, concludes: "Kolman's conception fully belongs to a type of research based on what is generally called 'the mechanist model of man'. And it is an integral part of a mechanistic, dehumanized model of science".
So far for one of the key ideologues on science and mathematics of the Stalin cult.

9.3.3. Dirk Struik

Dirk Jan Struik (1894 -2000) is a true human phenomenon. His mathematical works were first rate and when he visited the Netherlands to celebrate his 100th birthday, he gave in front of the black board, chalk in his hand, a very long entertaining talk about his trajectory in mathematics at Utrecht University {Albers, 2000}. On young age he joins the left wing split from the Social Democratic Party (SDAP), the SPD, which changed its name in 1918 in Communistische Partij in Holland (CPH), therewith becoming one of the oldest communist parties in the world.
Dirk and his brother Anton became and stayed committed communists. Anton belonged to the group of romantic engineers who travelled to the USSR to work on big engineering projects in Siberia and on the famous Siberia- Turkestan (Turksib) railway. Back in Holland

he played an important role in the party. He died due the “unintentional” bombing by the British air force of a ship full of German concentration camp prisoners from Neuengamme, May 1945.
Already in 1926, Dirk immigrated to the USA to start an academic carrier at MIT, in mathematics and in particular differential geometry. He was a founding editor of the magazine *Science & Society* and kept his political convictions till the end. Apart from the fact that Struik never in his long life ever mentioned the Moscow processes (and kept befriended with Kolman {Gordin 2017}, he is of interest because his “political” writings are mostly in the save space of the history of science and in particular mathematics. The same year he published his survey of Marx’ mathematical notes (see above) he published his *A concise history of mathematics* {Struik, 1948a, which has been updated and reprinted many times and is translated in many languages. His *Yankee Science in the Making* {Struik, 1948b} is a typical ‘historical materialist’ study on the early developments in his new country New England. The point I want to make is that books on the history of a field might well fit in the tradition of Hessen, Bernal, etc., but are in a way ‘neutral’ escapes from hard core DIAMAT publications (cf. Bernal) and hymns to the SU. So, let us look at works of Struik as a Stalinist ideologue and try to find out if and how Marxism, other than an historical materialist history writing, adds to the understanding of Mathematics.
In the first issue of *Science and Society* {Struik, 1936} he publishes an article ‘Concerning Mathematics’ which is primarily historical. “ In this interaction between theory and practice, between the social necessity to get results and the love of science for science’ sake, between work on paper and work on ships and in the fields, we see an example of the dialectics of reality, a simple illustration of the unity of opposites, and the interpenetrating of polar forms. It is a dialectic, not in the mind alone, but in the world of social realities, hence a materialist dialectics” (p.84). An open question is here if Struik considers thinking as a not ‘material’ human activity, or only the cranking of the material neurons? Diligently following Engels in discussing variable qualifies leading to the notion of a function, as was formalised in the west-European culture by people like Descartes and became “not only an example of the general dialectical law of the existence of the general in the particular, but a series of other higher important dialectical laws, such as the discovery of the continuous in the discontinuous, and the variable quantity in the static” (p.86). However: “This should not, however, be understood in the sense that mathematics before Descartes was undialectical. Engels himself observed that even the simplest mathematical relation show primitive dialectical structure” (id)”. Carefully he touches the issue of “what the forms are in which social causality exhibits itself in the history of mathematics” (p.89), and names important studies in the study of mathematics showing the role of societal influences. Interesting is how Struik quotes Kolman (and Yanovskaya on Hegel in Marx 1983) in comparing the constitution of the USA and the new draft constitution of the USSR, “as attempts, in two different forms of society, to give a definite formulation of the concept of freedom. Hegel, in breaking down the quantity-fetishism, already transcended philosophically the limits of capitalist society”. (p95). In a mixture of historical examples and adherence to Diamat, he stipulates: “The objects of mathematical study must be aspects of the objective world, existing independent of the human mind, which are gradually made a field for research........If ,.e.g., we state with Russell that mathematics is the class of all propositions

of the form "*p* implies *q*", we miss the relation to the objects of the real world to which our mathematical theorems all finally refer: to space and to number". (p.97). It becomes 'logical' that for an international renowned expert in differential geometry, his subsequent studies in the history of mathematics was a safer career choice than entering the discussion on the ontological or *a priori* quality of space and the 'dialectic' between inventions like complex numbers and their role in engineering, which Engels poked fun at.
In 1942 Struik publishes a most interesting and scholarly paper on the sociology of mathematics, dealing with the social contexts of developing mathematics, a subject in which he will become well appreciated {Struik, 1942}. He attacks correctly the then hegemonic style of historians of science of taking only the intrinsic developments of a field into account, as if scientific progress is an internal immanent development void of social interactions. A bit funny is the obligatory remark: "And, to give an example of recent times, dialectical materialism, far more than any of the past philosophies, has acted as a stimulant on the study of the exact sciences in the USSR, flourishing under the impact of socialist construction" (p.69). He wrote this in 1942, the year of the start of the Manhattan project and one year after the start of the Siege of Leningrad.
A long revision was published 44 years later {Struik, 1986}. Here almost all his political feathers are shaken off. The paper is a well written overview of Struik's own trajectory in the field of history of mathematics, from his historical materialist standpoint. Without going in any detail, he reviews the literature including old counsel-communist political adversaries such as Pannekoek and Sohn-Rethel and the inspiration he got from Bukharin's *Historical Materialism* (Russian edition 1921, German 1922). However, his easy going style fails to touch on any critique on the former discussions in the USSR, there with violating his own adage that all developments are dependent on the social-political circumstances and the paper presents an almost ripple free progress. He gives neither answer nor direction: "The question is, which aspects of modern mathematics are, or should be, cultivated in an industrial society of capitalist or socialist type, and which aspects in Third World countries?"(p. 298). At old age Struik lapsed into the role of a progressive researcher struck loose from his Diamat anchors. Interestingly, his lack of self-reflection in exemplified in footnote 7 on page 284 on one of the founding fathers of the historical materialist approach of the history of science Boris Hessen: "In 1935 he was unjustly arrested and died in prison. In 1956 he was posthumously rehabilitated. (Information received from E. Kelman (sic))". So far for what Struik ever said about all the Moscow processes. On Hessen see e.g.: {Graham, 1985}.
It is regrettable that this brilliant man in the last period of his life lapsed back to serious, but intellectually poor uncritical preaching of the historical place and value of the thinking of Marx and Engels, trying to prove that indeed historical materialism is correct that the progress of science is in function of the socio-economical political environment {Struik, 1992}. But no word or idea about the other side of the coin, how various - even immature- kinds of world views may induce other research avenues, which by themselves can play a role in shaping society. In other words, science remains something that seems to be "out there". New science influences the human discourse, like now the hype on Artificial Intelligence. However, Struik does not touch the question of doing science differently within a socialist world view. This issue is in line with the discussion in the young USSR and later

on China to catch up with capitalism by borrowing the hegemonic science & technology uncritically. In that sense Struik, like Pannekoek, Bernal, etc. kept in the confinement of established scientific thinking {see e.g. Kircz 2016}. Happily, new research on the issue how socialist thinking induced physics research is published {e.g. Kojevnikov, 1999, 2002, 2004}.
A short digression about Struik's political publications is interesting. After he obtained his US citizenship in 1934, he spends a year in Holland. At the same time that, together with his former tutor J.A. Schouten, professor at the Delft Polytechnic, he published his famous 2 volume *Einführung in die neuere Methoden der Differentialgeometrie* (1935;1938), he wrote under the pseudonym O. Verborg two substantial textbooks for the publishing house Pegasus, of the Dutch Communist Party, of which his brother was director. The first *Historical Materialism* {Struik 1935a} is a true catechism, complete with assignments after every chapter. It tells the true story of how it is. The style is easy and friendly without too many unsubstantiated jabs to others (as is common in this kind of texts) and with praise for tremendous changes in the SU, the successful first 5 year plan and Stalin's masterly description of the Sakhanov movement. He also stresses the continuity of the historical materialistic policies in the SU from Lenin to Stalin. Interestingly, he criticises the famous Dutch poet and left-communist theoretician Herman Gorter (1864-1927). Gorter wrote in 1908 a highly praised introduction: *Historical Materialism; explained for workers,* which was reprinted many times in Dutch, German, Swedish, and Russian. As Struik was an early member of the SPD in which Gorter played an important role, this booklet must have had a strong influence on him. After Gorter's sharp critique on Lenin and the young SU, Gorter became a dangerous enemy of the SU. Struik's critique is that Gorter divorces dialectal materialism form historical materialism, suggesting that historical materialism is not understandable without dialectical materialism, which suggest that Struik in line with his Russian masters says that theory is prior to historical experiment. According to Struik, Gorter also gives too much attention to Josepf Dietzgen (1828-1888), who at the time of Gorter's writing (1908) was very popular and was positively assessed by Marx, Engels and Plekhanov. As a matter of fact, Gorter doesn't deal with dialectics *per se* at all in this popular book.
Struik's second booklet is on Dialectics {Struik, 1935b} it is equally well written and heralds the dialectical methods as the panacea for all social and scientific problems of the world. With ample reference to Engels and Lenin and naming again Stalin's 5 year plan and his eminent thinking, the interesting upshot is that the dialectal method is presented as proof for the unification of all human knowledge in all fields. In fact the notion of dialectics is diluted to an abstract universal approach to human understanding in the sense that everything flows and everything is in continuous to and fro between the general and the particular, this way (actually against some currents in the SU) he sees quantum mechanics as a prove of the dialectics between particle and wave and relativity theory as the dialectical movement model for space, time and causality. Not a single discussion on ontological questions, even if Kant is -obviously- set apart as a bourgeois thinker, or on what the notions time and space mean. We see here most clearly, what we encounter in many "East-European" treatises, is in fact a pragmatic one-size-fits- all dialectical model, which serves not as a human invention, but as the essence of knowing per se, although not God's word.

Unfortunately, Struik, as US citizen and academic, and hence free from Stalinist bureaucratic interventions in his daily life and work, does not gives us any insight what dialectics means for mathematics beyond what Engels ever wrote. In fact the works are close cousins of {Adoratsky, 1934}, which original Russian version must have been published in the early ‘30s, and must be known by Struik. Interestingly, Struik repeats stupid critique on Deborin (Abram Moiseyevich Joffe 1881-1963), not a known comrade in The Netherlands; namely that studying dialectics is not enough, you have to practise it. This is in line with the anti-Deborin debate in the early 30th.[70]

In the next final section we will deal with a colleague, actually working in fields close to Struik's work, who tries to do better.

9.3.4. Aleksandr Danilovich Aleksandrov

A fascinating account of how to clinch to the wreckage of Diamat, while knowing better is given by the eminent mathematician Aleksandr Danilovic Aleksandrov (1912-1999), who got the Stalin price in 1942 and was as well, member of the Academy of science and party member.[71] After being the rector of the Leningrad State University, he joined in 1964 the Siberian Division of the Academy of Science of the USSR in Novosibirsk. [72]
In 1956 ADA was Rector of the University of Leningrad. It is also the year of the Hungarian revolution and the subsequent invasion of Hungary by soviet troops, the Suez Crisis and the famous 20th Congress of the CP, and the Poznan 1956 uprising. In other words, a year which turns out to be a hinge in world politics, especially for socialist countries.
Certainly for a loyal party man a pledge to the Diamat flag was part and parcel of the daily duties. Or to quote Alexander Vucinich {Vucinich 2000} after quoting two papers of 1951: “One cannot help but feel that Aleksandrov's defence of Leninism-Stalinism in Mathematics was only an adroit method for modulating Marxist views to make them less discordant with the ideas held by the community mathematicians. Aleksandrov argued, for example, that new mathematical knowledge must pass not only the test of logical integration into the existing body of theoretical knowledge and methodological procedures. New mathematical knowledge, he stated, was not necessarily as product of the scientific community's response to socially originated needs, for it could also generated by the internal dynamics of existing abstractions. He wrote: *The case of imaginary numbers and of non-Euclidean geometry show that the internal needs of mathematics- the needs for resolving abstract mathematical problems- may lead to significant conclusions and that the development of mathematics..... cannot be reduced to simple and direct reflections of nature because it includes formulations of far-reaching abstractions and theories”.*

---

[70] For Deborin see a.o.: http://www.sovlit.org/amd/index.html
[71] see: http://www.prometeus.nsc.ru/eng/science/schools/aleksad/
[72] for simple biographical notes see: http://www-history.mcs.st-andrews.ac.uk/Biographies/Aleksandrov_Aleksandr.html

All well and good, for a social- historians of science it is clear that the development of a field can have two driving forces, one the one hand the internal dynamics of a field, an aspect which was considered hegemonic in many circles even into the 20th century and the pressure of the surrounding social and productive forces, which became prime field of research as from the 1930s (see chpt X).
The issue is that the frozen static notion of Diamat as an all pervading superior method became indefensible in the course of modern science. This split forced career scientists in to the post-Stalinist drama of seeing M&E as Gods that failed.
Aleksandrov’s professional work proves his deep knowledge on the frontiers of geometry and hence the complicated discussions on e.g. the continuous vs the discrete. Although Aleksandrov wrote papers on philosophy and mathematics, not many of them are available and only a few translated in a language the present author is able to read. In reminiscence about the Siberian period of Aleksandrov, the mathematician Kutateladze {Kutateladze, 2005} pictures a positive story in which Aleksandrov is presented as an arrogant and volatile, but strong and scrupulous person. Remarkably, Aleksandrov shows a mixture of defending the party line as well as being stringent in cases of political anti-scientific bureaucratic meddling such as his support for the geneticists in their struggle against Lysenkoism, during his Rectorship in Leningrad.
So, what does Aleksandrov adds to our quest of Marxism and Mathematics. Certainly he was a geometer and as Kutateladze says: “The mathematics of the ancients was geometry (there were no other instances of mathematics at all). Synthesizing geometry with the remaining areas of the today's mathematics, Alexandrov climbed to the antique ideal of the universal science incarnated in mathematics” {Kutateladze, 2012}. Although it is difficult to square this with his remark that: ‘The general outlook of Alexandrov was determined by his scientific views that were formed in studying geometry. It is not by chance that the ideas of Karl Marx's Theses on Feuerbach enchanted Alexandrov”.

An example is the 1963 translation by the American Mathematical Society(AMS) of the important three volume textbook *Mathematics, Its content, methods and meaning* co-edited by A.N. Kolmogorov and M.A. Lavrent’ev {Aleksandrov, 1969}.
The first chapter is a general introduction to the history and art of mathematics without much ado. However, the original 1956 Russian edition knew two sections more: section 8-. *The essential nature of mathematics*, and 9- *The laws of development of mathematics*. Interestingly, the omission of paragraphs 8&9 in the translation was discussed in the AMS council in 1977 by Prof. Judy Green (later known for her work on women in maths) who considered it as a form of MacCarthyism, and anti-communist hysteria. Later these paragraphs where published in translation by the short living journal *Science and Nature* {Alexandrov, 1980}. Unfortunately, the editorial note does not inform us if the angry US supporters of Diamat ever had contact with Aleksandrov (who according to Kutateladze spoke excellent English) to check if his 1953 writings were induced by the horrible political situation of that time and if the author joined them in their anger anno 1980.

It is interesting to see how the struggle to cite the catechism and on the same time try to write textbooks on modern developments, becomes unconvincing 20 pages hotchpots of statements. As always the long quote from AD (see chpt F.Engels) that “Pure mathematics deal with space forms and quantity relations of the real world”. And “Engels arrives at the fundamental conclusion: mathematics has real matter as its subject, but considers it in complete abstraction from its concrete contents and qualitative peculiarities”, which is can indeed be taken as starting point. But based on this “allowed” abstractions statements about the most abstract mathematics contraptions are dealt with, without returning to the real world, apart from statements that the applied form of this abstract mathematics makes its inroads in e.g., physics. This impotent approach also leads to the viscous attacks on what is called, formalism, logicalism, and intuitionism schools in mathematics, as if the source of new inspiration coming from not productivist ideologies cannot lead to new proper science. Historically the opposite is often true as we see in the development of fields like the calculus, the infinite set theory, etc. Of course we can call it a scandal that the founder of infinite set theory, George Cantor, even wrote the pope ({Dauben, 1977} that his results are so encompassing that they must be an act of God, but it doesn’t help in disproving these results. Also Luzin’s inspiration by Christian mysticism does not devaluate his status as mathematician (for the Luzin affair see above).

The main problem is not that “Not himself a mathematician (FE), he was able to make such a profound analysis of this science not only because he was a thinker of genius , but mainly because he was able to use dialectical materialism, and was guided by it in his explanation of the essence of mathematics” (p31) ... It was exactly in this way that Lenin later gave an analysis of the problems in physics that surpassed anything done in this area” .Essential in this “non-religious?” thinking is that: “The truth of mathematical results is not, in the end, based on its definitions and axioms, not in the formal rigour of its proofs, but in real applications, i.e. in the final analysis, on practice”. (p35). Hence: “Thus, the difficulties of the development of mathematics under the conditions of capitalism beget and ideological crisis in this science, similar to the crisis in physics, the nature of which was explained by Lenin in his brilliant work *Materialism and emperio-Criticism*”.
Again it is important to note that Lenin in his fight for philosophical materialism clearly explained that ultimately a theory has to be find his validity in practise, (like Brouwer’s intuitionalism was a strong suggestion for algorithmic mathematics).
In particular, if we review the devastating prosecution in ‘30s-50s of scientists in many fields and considering Diamat and its social backlash on research and education, it is not strange that with the Diamat bathwater, the baby possibilities of novel approaches within an historical materialist framework became highly unpopular.
On the same level is the disgusting fact that after the implosion of Stalinism as a system, none of these thinkers were able to reflect on this episode and free themselves from the dogmatic straight jacket and try to launch a vigorous investigation how they could re-establish their “materialist” world outlook (see Struik above). It goes without saying that the last point is one of the prime reasons why e.g. the Orthodox Church could fill the moral, ethical and intellectual void in the former USSR. A permanent feature before the revolution, after the revolution and today is also the extreme nationalistic tone in claiming priority in novel developments.[73]

[73] E.g. Aleksandrov poses without any reference that the great Nikolai Ivanovich

In 1970, on the occasion of Lenin’s birth centennial, Aleksandrov published a long paper on Lenin: mathematics and dialectics {Aleksandrov, 1970}. The article starts with a long, but superficial and rambling history of mathematics in which quite a few observations can be criticized (e.g. that the discovery of the diagonal of a square, sqrt 2, was a break with empiricism, or that the idea of a ‘decisive experiment’ is tenet of physics, both p. 185). It would be better to say that the arithmetic expression of the diagonal was a break, because in pure land surveying the problem was not explicated, exemplified by the pure geometrical proofs of the theorem of Pythagoras.
In a way similar to that which material techniques extract different materials from nature, and transforms and combines them, affording man the means of mastering nature in practical work, so also mathematics is extracting from nature by means of abstracting its original concepts. Transforming and combines them, thus affording the means for the theoretical mastery of nature. The comparison with material technique accordingly suggests the definition of mathematics as “ideal technique’. The attempt to define mathematics as a tool and “‘...complement(s) the cognitive ability of man by its instruments and permits him to build theories of other sciences and to solve problems not accessible to the imagination or to direct thought” (p188)” sounds reasonable, provided we accept that material techniques don’t use abstract notions (or that all notions in technique are eternal reality), that people don’t learn to think in terms of -in other words make assessable to thought- e.g., radio waves.
“The concept of mathematics as ideal technique raises the further question of truth in mathematics. The difficulty here consists in the fact that the ideal objects of mathematics not only are not compared with those of reality, but the latter lack as well a sufficiently precise form. It is enough to recall the irrational numbers, not to speak of such matters as infinite sets of different powers. When we define axiomatically some objects of mathematics and talk runs to sets of objects of, 'arbitrary kind’, this appears to justify the aphorism of Russell that ‘mathematics is that doctrine in which we do not know whereof we speak nor whether what we say is true.’ However, the solution of the problem consists simply in knowing what the problem is not. Mathematics creates its apparatus and it is absurd to speak of what is true or false: the apparatus either works or it does not, and if it works it either works productively or poorly. It would be just as absurd to ask: "is this machine tool true or false?’ the machine tool simply is, and the meaningful question is whether it works in that for which it is suited. In this way, too, the ideal technique of mathematics with its apparatus simply is, it exists as a basic form of social activity and works in its sphere no worse than

---

Lobachevsky (1792-1856) routed his ideas in physics. Lobachevsky was with the German Johann Carl Friedrich Gauss (1777-1855) and the Hungarian János Bolyai (1802-1860), the inventor of non-Euclidean geometry. Gauss never published his work; Bolyai published his first work in 1832, whilst Lobachevsky published in 1829. Lobachevesky was his whole working life professor at Kazan University, whilst Bolyai did not found a serious career but left an enormous legacy of unpublished work. In the typical Russian nationalistic tradition and its obsession with priorities in science, Lobachevsky figures in many stories as the typical Russian hero. Obviously, this is difficult to square this with the historical materialist conception that new developments are produced when the time is ripe and ideas are ‘in the air’.

material technique. The question of truth arises only in applications of mathematics and the question depends not so much on itself as on what credence is given to its application” (p.189).

Obviously, this is a call for the independent integrity of mathematics, but also an unclear mixing between “true” in a mathematical logical sense, and “true” as a pragmatic way of saying “it works”. The last interpretation forgets that different mathematical approaches may lead to the same working. Another more positive interpretation can be that mathematics as seen by Aleksandrov is like our bike, a human invention that simply is. However, a bike is an invention that demands a certain societal infrastructure such as smooth roads, whilst mathematics is seen this way as something that certainly developed over the socio-historical periods but reaches a transcendental level free from social ties.

In his section on Dialectics and Mathematics, Aleksandrov states the old idea that the integration of “irrational, imaginary, impossible, not expressible in precise terms, not amenable to logic”,..”transformed to rational, real possible, expressible in precise terms and amenable to logic: and they were included in the system of mathematics not as heterogeneous, strange, contentious, but rather as an organically related, flourishing, living, and operating part”. He concludes that this process of unification in mathematics into the present encompassing theory (that is say set theory) is a result of the internal dynamics of mathematics and: “This logic -the logic of changing concepts in accord with the problem of cognition- is dialectics” (p.194). Having said that, it simply means that dialectical thinking is not a driving force but is the expression of the fact that ‘correct’ mathematics in which simple oppositions such as rational-irrational numbers, or between discrete and contiguous are ‘solved’ by a broader theory. “In other words, ideas that are seemingly antagonistic, and are subsequently represented in a new theory or form are proof for the dialectics.
Or ...dialectics, the logic of the apprehension of new concepts, logic in particular, forming and generalizing investigation of the instruments, the concepts, the formal theories of mathematics”.
Followed by the obvious quote from Lenin in his comments in Hegel’s logic ‘
“All-sided, universal flexibility of concepts, a flexibility reaching to the identity of opposites,—that is the essence of the matter. This flexibility, applied subjectively =eclecticism and sophistry. Flexibility, applied objectively, i.e., reflecting the all-sidedness of the material process and its unity, is dialectics, is the correct reflection of the eternal development of the word “ {Lenin VW 38, p.110}l (p.195).

The upshot is that Aleksandrov following Engels (and Hegel) posits that every unity of opposition is an expression of dialectics. The key issue is here; if we can accept the idea that a (ideal) mathematical opposition such as the discrete and the continuum, or rational and irrational numbers have the same dialectical ‘value’ in dialectical materialism as the antagonism between workers and capitalists class? As Aleksandrov himself argues in modern mathematics the oppositions are overcome. The consequence of this idea is the possible claim that mathematics is now an internal consistent theory, void of possible new oppositions. It would be the same as claiming that socialism has been reached in the SU. If

indeed we could apply the notion of dialectics to mathematics than new oppositions and antagonisms in the present stage of mathematics must come to the fore. An impossible figure as long as mathematics adhere to formal logic, which in and by itself in its proof theory is build up from tautologies and modes ponens reasoning. Reducing dialectical logic to the whole trinity is not enough. [74]
A year later Aleksandrov write another paper on Mathematics and Diabetics {Aleksandrov, 1971}. Obvious there are many overlaps with the paper discussed above.
After some historical remarks about the solving of 4 puzzles (negation of the parallel postulate, imaginary numbers, algebraic functions, and infinitely small), in the 19th century, all to be cleared up in set theory.

By the development of new branches in mathematics, that failed to become unified in one all-encompassing axiomatic system (as was Hilbert's research programme) we see a new phase in mathematics by the emerging of such as algorithm, automata, game, etc. theory. "In diesen Theorien wird mit der gleichen der Mathematik eignen Form der absolutierten Idealisierung vor allem die Tätigkeit des Menschen selbst erforscht, und zwar die Möglichkeit der mathematischen Herleitung und Lösung von Aufgaben durch jeweils vorgegebene Mittel, die Informationsübertragung, die steuerung u.a. In diesem Sinne ist der Mathematik zu einen Humanwissenschaft geworden'. Durch das Auftauchen von mathematischen Maschinen wurde dann die Mathematik zu einer technische Wissenschaft(p.53) ....
So: „Die formelen Theorien sind an and fur sich selbst Strukturen, jedoch dienen andere Strukturen, die in die Sphäre der Mathematik überhaupt eingehen und mit einen oder anderen Grad an Gehalt und auf dem einen oder anderen Abstrraktionsniveau verstanden werden, als Gegenstand der Formalisierung oder zur Interpretation der formalen Theorien (p.53-54)."
In other words, Aleksandrov sees Mathematics as a formal (ideal) tool for the theoretical mastering of nature on par with experimental tools. But: "Die Mathematik entwickelt sich als universelles Mittel jeglicher Wissenschaft. ...
Welchen Entwicklungsaspekt der Mathematik (die axiomatische Methode, die Diskretheit und Kontiguität, das Endliche und Unendliche) wir auch konkret untersuchen würden , ins Auge fällt immer ihrs allgemeines Merkmal - die Feststellung 'der Identität der Gegensatze". (p.59).

Summing up, one can say that Aleksandrov's pursuit is an attempt to prove that modern mathematics is dialectical by and in itself. He illustrates this with the traditional notions starting with the example that negative numbers are oppositions to natural numbers etc. The solution to this opposition is the invention of zero, which enable us to see all integer numbers as one collection. This reasoning leads to the notion that within mathematics we

[74] Interestingly contrary to older work the founders of the formalistic school (Hilbert) and intuitionism school (Brouwer) and others are more positively named as one sided ingredients to the development of mathematics. To say: "to the intuitionist, mathematical truth is in the mathematician's head, while to the formalist, it is on paper" (p. 187) is a nice quip, but does it mean that his mathematics can be equated with "objective reality"?

are able to include ‘opposing’ notions like discrete and continuous into a new formal system e.g. set theory in which the unity of oppositions is vested. In that sense mathematics indeed becomes equal to a technical tool. After all it works. In Marx words “from the concrete to the abstract and back to the concrete”, but the dynamics of changing content of notions to cater for this option is weak. Is it indeed that simple that we can play with oppositions as Engels did and call imaginary numbers in opposition to real numbers, or is this not more a result of immature science? In formal ideal systems it is easy to define opposite notions, but if the result is an extension of the basic notions it becomes cheap. To say that real rational numbers are the quotient of natural numbers does not mean that numbers that are not quotients of rational numbers are in opposition to rational numbers. The same question holds for more complicated notions as the discrete and the continuous. The identity of oppositions is only one aspect of the dialectics and the most easy and formal. The idea of transcending oppositions onto novel levels of understanding is not (dis)solved by reducing the notion of a continuum to an infinite collection of discreet numbers.

9.3.6. And finally Mao

The historian of mathematics Dauben {2003, 2004} tells us the story how ideological tyranny can be circumvented by referring to higher powers.
As mentioned earlier, in the discussion on the calculus, we have originally seen two approaches. Newton’s, who dealt with an ever more refined cutting of a line (the secans) through a curve representing motion, and Leibnitz, who proposed infinity small entities -infinitesimals- to calculate the derivative. Marx was, as many others in his time, concerned with the possibility of finally reaching a situation where we have to deal with zero divided by zero, which is ill defined. For this reason as well as the notion Engels defended that counting was the foundation of mathematics, many Marxists preferred the -atomistic- Leibnizian approach as there we deal with countable -very small- units. We have discussed above already that this problem was solved by a redefinition of the notion ‘number’ and the introduction of the notion limit. However, from a fundamental point of view the idea of infinitesimals as mathematical acceptable objects stayed on the agenda. One fundamental notion here is that “normally” in analysis if we multiply a number with a large other number the result will be larger than any other given number. This is called Archimedean property. In 1966 the American mathematician Abraham Robinson developed a consistent theory of non-Archimedean infinitesimals named non-standard analysis.

During the Cultural Revolution in China abstract mathematics was condemned. The publications of Marx’s mathematical manuscripts lead to the rehabilitation of mathematics. So in early 1970's two groups, one in Beijing and one in Shanghai started a Chinese translation, and published them with great fanfare. “The promote the great campaign criticizing Lin Biao and Confucius, the *Mathematical Manuscripts* of [Karl] Marx, who inspired the proletarian revolution, were translated and edited by the Mathematical Manuscript Study Group of Beijing University, and published by the People’s Press. This is a great event in our ideological battlefield” {translation from the Chinese by Dauben, 2004, p 307}. An enthusiastic discussion commenced in the Chinese mathematical community, spiced with Mao quotations about the importance of popularizing science. In 1976 also Robinson's approach reaches the Chinese mathematicians and became an important ingredient in discussion Marx's critique on the calculus.

To quote Dauben {2003, 355-6}: the mathematician "Shu Ji devotes an entire section of his article to arguing that "the infinitely small (large) really are real numbers", with the "reals are real numbers" meaning here that they are ontologically real, concrete—in physical, material terms. After quoting from Marx's Mathematical Manuscripts, Engels' Dialectics of Nature, and Chairman Mao's essay "On the correct handling of contradictions among the people", Shu Ji claims that: "Robinson himself recognized that nonstandard analysis was grounded in a concrete, material way in so far as the usefulness of infinitesimals was best seen in applications to real-world problems (Shu Ji 1976, p. 2)". So as a result of the authority of Marx and Mao, the first symposium on nonstandard analysis was held in August 1978 in Xinxiang, Henan Province. {Dauben, 2003, p. 359}.

Unfortunately, nor in China, nor in the SU, or the West, nonstandard analysis became mainstream and certainly not in ideological battles. We have no idea how its fits in the schema of Aleksandrov. But the understanding of infinitesimals, which do not accord with the Archimedean principle (you can multiply them with any number, but they remain infinitesimals) and the wider notion of Hyper numbers, is just another proof that although people have ten fingers, which did not have led to our present counting system as Engels suggests, human thinking is going beyond this concrete example and in its abstraction might return in concrete mathematical puzzles.

Bibliography

A GREAT NUMBER OF TEXTS BY MARXISTS CAN BE FOUND ON THE repository: https://www.marxists.org/ HOWEVER, A WEB PRESENTATION DOESN'T HAVE ALWAYS PAGENUMMERS GIVEN. Hence I prefer to refer to the paper version, if available.

Adoratsky 1934 V.Adoratsky, *Dialectical Materialism; the theoretical foundation of Marxism-Leninism*, Martin Laurance, Ltd, 1934.

Alberts (2000).
Gerard Alberts, 'In memoriam Dirk Struik (1894-2000), *Nieuw Archief voor Wiskunde (NAW)*, 5/1 nr. 4 december 2000, pp 354-355..

A.D. Aleksandrov, A.N. Kolmogorov and M.A. Lavrent'ev, 1999
Mathematics, Its content, methods and meaning, AMS 1963, MIT Press 1963, Dover publication 3 volumes in one, 1999.

Aleksandrov
1980. Mathematics its essential nature and objective laws of development, *Science and Nature: A Journal of Marxist Philosophy for Natural Scientists* (#3, p 22-42) and put online at:
http://www.massline.org/Philosophy/Math/Mathematics-CensoredMarxistPhilosophy.pdf .

1970. A.D.Alexandrov (1970), On the one hundredth anniversary of the the birth of V.I. Lenin: mathematics and dialectics. Translated from *Sibirskii Matematicheskii Zhurnal*, Vol.ll, No.2, pp.243-263, March-April, 1970. Consultants Bureau, a division of Plenum Publishing Corporation.

1971. A.D.Alexandrov (1971). Alexander Alexandrow. Mathematik und Dialektik, *Ideen des exacten Wissens*, Heft 4, April 1971, s. 251-257. Published in: Michael Otte (ed.) *Mathematiker über die Mathematik*, Springer Verlag 174, pp. 47-63.

Alcock (2014).
Alcock, Lara, *How to think about analysis*, Oxford UP, 2014

Alcouffe (2000)
Alain Alcouffe & Julian Wells (2009), *Marx, maths, and MEGA2*, presented at the 13th Annual Conference of the European Society for the History of Economic Thought University of Macedonia, Thessaloniki, Greece, Apr 2009,
Available on: https://www.researchgate.net/publication/318820526_Marx_maths_and_MEGA_2

Anellis
1979 Irving H. Anellis, 'Review of van Heijenoort, *With Trotsky in Exile", Studies in Soviet Thought*, 20 (1979), pp 391-396.

1988 Irving H. Anellis, Jean van Heijenoort, the revolutionary, the scholar, and the man (1912-1986), *Studies in Soviet Thought*, 35 (1988), pp 147-178.

1996 Irving H. Anellis (1996), Sof'ya Alexsandrovna Yanovskaya's contribution to logic and history of logic, *Modern Logic,* Vol 6, no.1. 1996, pp 7-36.

2012 Irving H. Anellis (ed.) Special Issue: Perspectives on the History and Philosophy of Modern Logic: Van Heijenoort Centenary, *Logica Universalis* Volume 6, Issue 3-4, December 2012.

Bailes 1978.

Kendall E. Bailes, *Technology and society under Lenin and Stalin; origins of the of soviet technical intelligentsia,* Indiana UP, 1978.

1977 Kendall E. Bailes, Alexei Gastev and the soviet controversy over Taylorism, 1918–24, *Soviet Studies*, vol.XXIX, no.3, July 1977, pp. 373-94.

Bazhanov 2001.

Valentin A. Bazhanov, ‘Restoration: S.A. Yanovkaya’s path in logic’, *History and philosophy of logic*, 22:3, 129-133.

Bell

2005 John L. Bell, *The Continuous and the Infinitesimal in Mathematics and Philosophy*, Milan: Polimetrica S.A. Also available at: http://publish.uwo.ca/~jbell/The%20Continuous.pdf

2017 John L. Bell, *Continuity and infinitesimals*, *The Stanford Encyclopedia of Philosophy* (Summer 2017 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/sum2017/entries/continuity/>.

Van Benthem,

2010 Johan van Benthem, *Modal logic for open minds*, CSLI publications, 2010.

Boer

2014 Roland Boer; *In the vale of tears.n Marxism and theology*, Vol.5, Haymarket Books 2014.

2017 Boer, Roland, 2017, Roland Boer, *Stalin: From Theology to the Philosophy of Socialism in Power* Springer

Bogdanov

2016 A.A. Bogdanov, *The Philosophy of living Experience*, Historical materialism series 111, Bogdanov Library 8, Brill 2016. First Russian edition 1923.

Boyer 1949.

Carl B Boyer, *The history of the calculus and its conceptual development*. Dover edition 2017.

Florian Cajor (1928-9)

A history of mathematical notation, Two volumes bound as one, Dover, 1993

Campbell 1961.
Robert W. Campbell, Marx, Kantorovich, and Novozhilov: Stoimost' [value-jk] versus Reality, *Slavic Review*, Vol. 20, No. 3 (Oct., 1961), pp. 402-418

Carchedi
2008 Guglielmo Carchedi, Dialectics and Temporality in Marx's Mathematical Manuscripts, *Science & Society*, Vol. 72, No. 4 (Oct., 2008), pp. 415-426

2008b Guglielmo Carchedi, Logic and dialectics in social science, *Crit Sociol September 2008* vol. 34 no. 5 631-656

2011 Guglielmo Carchedi, *Behind the Crisis*, *Marx's dialectis of value and knowledge*, Brill. Paperback Haymarket Books 2012.

2014 Guglielmo Carchedi, The crisis and Marx' theory of knowledge, In: Sara R. Farris, *Returns of Marxism; Marxist theory in a time of crisis*, IIRE, reprinted. Haymarket Books, 2016.

Christopher A. J. Chilvers, 2015: Five Tourniquets and a Ship's Bell: The Special Session at the 1931 Congress, *Centaurus*, 2015: Vol. 57: pp. 61–95;

Courant-Robbins 1996.
Richard Courant and Herbert Robbins, Revised by Ian Stewart, *What is mathematics? An elementary approach to ideas and methods*, Oxford UP, second edition, 1996.

Damsma, 2015.
Dirk Folkert Damsma, *On the articulation od systemic-dialectical methodology and mathematics*, PhD Thesis, Univ. of Amsterdam.
Available on: https://pure.uva.nl/ws/files/2326599/154609_DAMSMA_thesis_met_cover.pdf

Dauben
1977. Joseph W. Dauben, Georg Cantor and Pope Leo XIII: Mathematics, Theology and the Infinite, *Journal of the History of Ideas*, Vol. 38, No. 1 (Jan-Mar), pp. 85-108.

2003 Joseph W. Dauben, Mathematics, ideology, and the politics of infinitesimals: mathematical logic and nonstandard analysis in modern China, *History and Philosophy of Logic*, 24:4, 327-363,

Demidov 2016
Sergei S Demidov and Boris V. Lëvshin (eds), Roger Cooke (transl.), *The Case of Academician Nikolai Nikolaevich Luzin*, Hisotry of Mathematics vol. 43, American Mathematical Society, 2016. Original Russian publication: RHGI, 1999.

Derschowitz and Reingold 1997
Nachum Derschowitz and Edward M. Reingold, *Calendrical Calculations,* Cambridge UP, 1997.

Dijkgraaf
2018a. Robbert Dijkgraaf, "Afgunst op de fysica is zó vorige eeuw' , , *NRC-Handelblad* Section science. 24-25- Februari 2018, p.w2.

2008b. Robbert Dijkgraaf, 'Het hebgelijkteken', *NRC-Handelblad* Section science and education, 15 March 2008, p.38.

Engels
1886. F. Engels, *Ludwig Feuerbach and the End of Classical German Philosophy*, *MECW*, Vol.26.

1878. F. Engels, *Anti-DÜHRING, Herr EUGEN DUHRING'S revolution in science,* MECW, Vol. 25. pp. 5- 312

(1873-1883).
Dialectics of nature, MECW, vol 25, pp 313-590.
1985. Friedrich Engels , 1985, *Dialektik der Natur (1873-1883)*, MEGA2, Dietz Verlag.

1884. F. Engels, *The Origin of the Family, Private Property and the State. In the Light of the Researches by Lewis H. Morgan*, MECW Vol 26, L&W, 2010.

Feferman 1993.
Anita Burdman Feferman, *Politics, Logic, and Love: The Life of Jean van Heijenoort,*, A K Peters/CRC Press, 1993.

Ford 1997. Charles E. Ford, 'Mathematics and Marxism', in: Eberhard Knobloch, Jean Mawhin and Serguei S. Demidov (eds.). *Studies in history of mathematics dedicated to A.P. Youschkevitch; Proceedings of the XXth International Congress of History of Science (Liège, 20-29 July 1997)*, Brepolis, 1997.

Gare 2000 . Gare, Arran, 'Aleksandr Bogdanov's history, sociology and philosophy of science' *Stud. Hist. Phil. Sci*. 31,2: 231 -248.

Gerovitch 2001.
Slava Gerovitch, 'Mathematical Machines' of the Cold War: Soviet Computing, American Cybernetics and Ideological Disputes in the Early 1950's, *Social Studies of Science*, Vol.31, no.2 , Science in the Cold War, April 2001, pp 253-287.

Gnedenko 1970.
B.V. Gnedenko, V.I. Lenin and methodological questions in mathematics, *Russian Mathematical Surveys*, vol .25, no 2. 1970. This journal is the translation of the Russian *Uspekhi Matematicheskikh* Nauk, published by the British Institute of Physics.

Graham

1985. Loren R. Graham, “the socio-political roots of Boris Hessen: Soviet Marxism and the history of science’, *Social Studies of Science*, vol 15 (1985), 705-22.

1987. Loren R. Graham, *Science, philosophy, and human behavior in the Soviet Union*, Columbia Univ. Press, 1987.

2009. Loren Graham and Jean-Michel Kantor, *Naming Infinity; A True Story of Religious Mysticism and Mathematical Creativity*, The Belknap Press of Harvard UP, 2009.

Greene,

Brian Greene, *The elegant universe; superstrings, hidden dimensions, and the quest for the ultimate theory*, Vintage Books, 2003.

The fabric of the cosmos; space, time, and the texture of realit, Penguin books, 2004.

Gordin 2017.

Michael D. Gordi, The Trials of Arnošt K.: The Dark Angel Of Dialectical Materialism, *Historical Studies in the Natural Sciences*, Vol. 47, Number 3, pp. 320–348

Harman 1982.

P. M. Harman, *Energy, Force, and Matter. The conceptual development of nineteenth-century physics*, Cambridge UP, 1982.

Stephen Hawking , 1988,

A brief history of time; from the big bang to black holes, Bantam

Van Heijenoort

1948. Jean van Heijenoort (1948), (Jean Vannier), ‘A Century’s Balance Sheet’, in: *Partisan Review*, Vol.15 No.3, March 1948, pp.288-296. https://www.marxists.org/history/etol/writers/heijen/1948/03/balancesheet.html

1978. Jean van Heijenoort, *With Trotsky in Exile, from Prinkipo to Coyoacán*, Harvard UP.

1985. Jean van Heijenoort: *Selected Essays*. History of Logic III, Bibliopolis, 1985.

Don Howard, 2004,

“‘Let me briefly indicate why I do not find this standpoint natural.’Einstein, General Relativity, and the Contingent A Priori.” Invited lecture delivered at the conference, “Synthesis and the Growth of Knowledge: Examining Michael Friedman’s Approach to the History of Philosophy and Science,” University of South Carolina, 1-3 October 2004. Available on: https://www.researchgate.net/publication/240632718_Let_me_briefly_indicate_why_I_do_not_find_this_standpoint_natural_Einstein_General_Relativity_and_the_Contingent_A_Priori

Ifrahm (1994).

Georges Ifrah, *The universal history of numbers*. Vol.1, *The world first number -systems*; Vol.2, *The modern number- system;* Vol.3, *The computer and the information revolution,* The Harvill Press, 2000. (Translated from the French).

Ilyenkov (1982).

Evald Ilyenkov, *Reflections on Lenin's Book: "Materialism and Empirio-Criticism"* ,1979. English translation: *Leninist dialectics and the metaphysics of positivism*, New Park Publications, 1982.

Jammer

1961. Max Jammer, *Concepts of Mass in Classical and Modern Physics*, Harvard UP, Reprinted as Dover Publication Inc. 1997.

1999. Max Jammer, *Concepts of force,* with a new preface to the 1957 Harvard UP edition, Dover.

2000. Max Jammer, *Concepts of Mass in contemporary physics and philosophy*, Princeton UP.

2006. Max Jammer, *Concepts of simultaneity, from antiquity to Einstein and beyond*, The John Hopkins UP, 2006.

Jensen (1978).

Jensen, K.M. 1978. *Beyond Marx and Mach*, Dordrecht: D. Reidel Publishers Co.

Joravsky (1961).

David Joravsky, *Soviet Marxism and natural Science*, *1917-1932*, Columbia Univ. Press.

Kaparov (2018).

Garry Kasparov, *Deep Thinking, where artificial intelligence ends and human creativity begins,* with Mig Greengard, John Murray publ.

Kircz

2015 Joost Kircz, **Reality, knowledge and forecasting**.
Essay based on my presentation at the conference: *Le troisième âge du capitalisme, sa physionomie socio-politique à l'orée du XXIe siècle. En mémoire d'Ernest Mandel (1923-1995),* May, 20-22, 2015, Lausanne, Switzerland.
English version, as IIRE Working paper 41., 27 pages.
Also in French. http://www.kra.nl/Website/ArtkelenSP/Lausanne-Kircz-Trad-28-3-17.pdf

2016. Joost Kircz, *Science: model, icon, and the subjective factor*, presentation on the 13th Annual Historical Materialism Conference: Limits, Barriers and Border, SOAS, London, 9-13 November 2016. Available at:

http://www.kra.nl/Website/ArtkelenSP/Pannekoek-Bernal-Fin.pdf

Koestler (1949).
Arthur Koestler, Igmazio Silone, Richard Wright, André Gide, Louis Fisher, Stephen Spender, Richard Crossman (editor), *The God that failed, a confession*, Harper, 1949.

Kojevnikov
1999. Alexei Kojevnikov, “Freedom, collectivism, and the quasiparticles: social metaphors in quantum physics”. *Historical studies in the physical and biological sciences*, 29 (2), pp. 295-331

2000. Alexei Kojevnikov, *Games of Stalinist Democracy; Ideological discussions in Soviet science 1947-52*, In: Sheila Fitzpatrick (ed.), *Stalinism, new directions*, Routledge, 2000, pp. 142- 175. This a slight revision of the paper ‘Rituals of Stalinist Culture, *Russian Review*, 57, 1(January 1998, pp 23-52)

2002. Alexei Kojevnikov, “David Bohm and collective movement”, *Historical studies in the physical and biological sciences*, pp. 161-192.

2004. Alexei B. Kojevnikov, *Stalin’s great science. The times and adventures of Soviet Physicists*, Imperial College Press.

Kolman.
1930 Kol'man, E. "Protiv nauchnoi mistifikatskii v oblasti planirovania (politika, ekonomika i... matematika)." Pravda, October 31, 1930.

1935. E. Kolman, On the problem of a unified physical theory of matter, *Philosophy of science*, vol.2. no 4 (oct. 1935) pp. 400-412.

1965, E. Kolman. Arnost Kolman, *Considerations about the certainty of knowledge*, Robert S. Cohen & Dirk J. Struik (eds.), Occasional paper no.2, The American Institute for Marxist Studies (AIMS) , 1965. 12 pp.

1968. E. Kol’man: Karl Marx and Mathematics: on the ‘Mathematical Manuscripts’ of Marx, In Marx 1983, pp. 217-255.

1982. Arnoŝt Kolman, *Die veriirete Generation; so hätten wir nicht leben sollen, Eine Autobiographie.* Including: *Wie habt ihr so leben können? Ein Dialog zwischen Františ̂ek Janouch und Arnoŝt KolmanI,* H. Haefs u. F. Janouch (eds), Fischer Taschenbuch Verlag.

Kolman & Yanovskaya (1931), in: Marx, 1983, pp. 235-255

Kots and Seneta (1990).
Samuel Kotz and Eugene Seneta, ‘Lenin as a statistician: A non-Soviet view’, *Journal of the Royal Statistical Society, Series A (statistics in society)* Vol.2153, no.1, pp: 73-94.

Kovály (1972).

Kovály Pavel, Arnošt Kolman: Portrait of a Marxist-Leninist philosopher, *Studies in Soviet Thought*, 12, pp. 337-366.

Kniga1931. *Science at the cross road, papers presented to the International Congress of the History of Science and technology held in London from june 28th to July 3rd, 1931 by the delegates of the U.S.S.R.*, Kinga (England) Ltd., 1931.

Alexei Kojevnikov 1999, "Freedom, collectivism, and the quasiparticles: social metaphors in quantum physics". *Historical studies in the physical and biological sciences*, 29 (2), pp. 295-331

Alexei Kojevnikov 2002, "David Bohm and collective movement", *Historical studies in the physical and biological sciences*, vol. 33, pp. 161-192.

Kula, 1986, Witold Kula. *Measures and Men*. Princeton UP1986

Kutateladze

2005. Semen S. Kutateladze, Reminiscences about Alexandr Danilovich Alexandrov (1912--1999). Submitted on 11 Jul 2005 to: https://arxiv.org/abs/math/0507204 also in: *Siberian Electronic Mathematical Reports*, Vol. 9, A44--A61.

2012. S.S. Kutateladze, Alexandrov of Ancient Hellas, On the Occassion of the Centenary of the Birth of Aleksandr Alexandrov (1912–1999). *Siberian Electronic Mathematical Reports*, Vol. 9, A6–A11 (2012). Available: http://www.math.nsc.ru/LBRT/g2/english/ssk/talk_ad100_e.html

Lafargue (1890).
Paul Lafrague, "Reminiscences of Marx", in: *Marx and Engels Through the Eyes of Their Contemporaries*, Progress Publishers, 1972. Also at: https://www.marxists.org/archive/lafargue/1890/xx/marx.htm

Lakoff and Núñez (2000).
George Lakoff & Rafael E. Núñez. *Where mathematics comes from. How the embodied mind brings mathematics into being*. Basic Books.

Lecourt (1975).
Dominique Lecourt, *Une crise et son, enjeu essai sur la position de lénine en philosophie*, Maspero, 1973. I use the German translation by Ursel Rütt-Förster: *Lenins philosophische Strategie; Von der Widerspiegelung (ohne Spiegel) zum Prozess (ohne Subject).* Ullstein Buch, nr 3207, 1975.

Lenin
1908. Lenin (Vladimir Ilyich Ulyanov). *Materialism and Empirio-Criticism. Critical*

comments on a reactionary philosophy (1908), Lenin, *Collected works*, vol. 14, Progress Publ. 1968.

1972. *Philosophical Notebooks*, Lenin CW vol. 38. Progress Publ. 1972

Marcel van der Linden, 2008,
Workers of the World. Essays toward a Global Labor History, Leiden and Boston: Brill, 2008, chapter 2 and 3

Lindley (2010).
Mark Linsey, Marx and Engels on Music in *MR online*, posted Aug.18, 2010. https://mronline.org/2010/08/18/marx-and-engels-on-music/

Lorentz (2002).
G.G. Lorentz, Mathematics and Politics in the Soviet Union from 1928 to 1953, *Journal of Approximation Theory*, 116, pp.169-223.

Marx
1875. K. Marx, Critique of the Gotha Programme. MECW V.24.

1976 Karl Marx. *Capital vol 1. 1867-1886* (Trans. Ben Fowles, intro EM) Penguin, 1976

1967. *K. Marx, F. Engels, V. I. Lenin On scientific Communism*, Progress Publishers.

1983. Sofya A, Yanovskaya and Ersnt (Arnost) Kolman ed: *Mathematical manuscripts of Karl Marx*, First published in German and Russian, Nauka Press, 1968. An English Translation: New Park Publications, 1983

1994. Shri Pradip Baksi editor, *Karl Marx Mathematical Manuscripts. Together with a Special Supplement*, VISWAKOS PARISAD (Calcutta).

Mirowski 2011.
Philip Mirowski. *Science-Mart, privatizing American Science*, Harvard UP.

Moore 1990.
A.W. Moore, *The infinite* Routledge.

Muller 2009.
Karl Muller, Zur Entstehung und Wirkung von Engels “Anti-Dühring”, Trend - Onlinezeittung, 09/04 : http://www.trend.infopartisan.net/trd0904/t010904.html

Mark Ovenden, 2005, *Metro maps of the world*, Capital Transport Publ.

Jürgen Renn, 2020,*The evolution of knowledge; rethinking science for the Anthropocene*, Princeton UP.

Schmandt-Besserat (1992).
Denise Schmandt-Besserat, *How Writing Came About*. Univ. of Texas Press, Austin, Texas.

Siegmund-Schultze (2004).
Reinhard Siegmund-Schultze, 'Mathematicians forced to philosophize: An introduction to Khinchin's paper on von Mises' theory of probability', *Science in Context*, 17 93), pp. 373-390.

Seneta (2004).
Eugene Seneta, Mathematics, Religion, and Marxism in the Soviet Union in the 1930s, *Historica Mathematica*, 31, pp337-367, 2004.

Lee Smolin 2007
The trouble with physics; The rise of string theory, the fall of a science, and what comes next, A Mariner Book, Houghton Mifflin Co.

Smolinski,

1964 Leon Smolinski, What Next in Soviet Planning? : *Foreign Affairs*, Vol. 42, No. 4 (Jul., 1964), pp. 602-613

1973. Leon Smolinski, Karl Marx and Mathematical Economics, *Journal of Political Economy*, Vol. 81, No. 5 (Sep. - Oct., 1973), pp. 1189-1204

Stalin
1953. J.V. Stalin, *Marxism and problems of linguistics*, June 20, July 4, and August 2, 1950 issues of Pravda and: https://www.marxists.org/reference/archive/stalin/works/1950/jun/20.htm

Van Stigt 1990.
W.P. van Stigt, *Brouwer's Intuitionism*, Studies in the history and philosophy of mathematics, Vol.2. North-Holland, 1990.

Struik
1935a. (Dirk J. Struik). O. Verborg: *Het Historisch Materialsme*, Pegasus. A second reworked edition appeared in 1937.

1935b. (Dirk J. Struik). O. Verborg: *De wereldbeschouwing van het communisme. Het dialektisch materialisme*, Pegasus, 1935.

1936. Dirk J. Struik, Concerning Mathematics, *Science & Society*, Vol.1, no.1 (fall, 1936) pp 81-101.

1942. Dirk J. Struik, On the sociology of Mathematics, *Science & Society*, Vol.6 No.1 (winter, 1942), pp. 58-70.
1948a Dirk J. Struik, *A concise history of Mathematics*, Dover,

1948b Dirk J. Struik, *Yankee Science in the Yankee Science in the Making: Science and Engineering in New England from Colonial Times to the Civil War*, Lyyle, Brown and Co, (1st ed.).

1948c Dirk J. Struik, Marx and Mathematics, *Science and Society*, vol.12, no. 1, Centenary of Marxism (winter, 1948), pp.181-196.

1986. Dirk J. Struik, The sociology of Mathematics Revisited: A personal Note, *Science & Society*, Vol 50, No.3. Special 50th Anniversary Issue (Fall, 1986), pp. 280-299.

1992. Dirk J. Struik, Marx and Engels on the history of science and technology, In: Sergei S. Demidov, David Rowe, Menso Fokerts and Christoph J. Scriba (eds), *Amphora: Festschrift für Hans Wussing zu seinen 65. Geburtstag*. Birkhäuser, 1992. In a note it is said;" This paper was written in 1966, but it does not seem dated too much".

Susiluoto (1982).
Susiluoto, Ilmar. *The origins and development of system thinking in the Soviet Union; Political and philosophical controversies from Bogdanov and Bukharin to present-day re-evaluations*, PhD thesis, Helsinki: Annales Academiae Scientiarum Fennicae Dissrtationes Humanurum Litterarum 30, Suomalainen Tiedeakatemia.

Vucnich (2000).
A.Vucnich: Soviet mathematics and Dialectics in the Stalin Era, *Historia Mathematica*, 27, p.27

Weber, 1893.
Kronecker made this remark in a presentation of the Berliner Narturforscher-Versammlung in 1886. H. Weber. Leopold Kronecker, Jahresbericht der Deutschen Mathematiker-Vereinigung, Zweiter Band, 1891-92, p. 19.
http://www.digizeitschriften.de/dms/toc/?PID=PPN37721857X_0002%7Clog2

Werskey 1988.
Gary Werskey, *The visible college: a collective biography of British scientists and socialists of the 1930s*, Free Association Books, 1988.

Wiener 1948.
Norbert Wiener, *Cybernetics, or Control and Communication in the Animal and the Machine,* MIT Press.

Wigner 1960.
Eugene P. Wigner, The Unreasonable Effectiveness of Mathematics in the Natural Sciences, *Communications on pure and applied mathematics*, Vol. XIII, 001-14.

Frank Wilckek 2015
A Beautiful question; finding nature's deep design, Penguin Press

Peter Woit, 2006

Not even wrong; the failure of string theory and the search and the search for unity in physical law , Basic Books

Yakhot 2012.
Yehoshua Yakhot, *The suppression of philosophy in the USSR (the 1920s & 1930s)*, Mehring Books.

S.A. Yanovskaya (1968) , PREFACE TO THE 1968 RUSSIAN EDITION, In: Marx 1983, pp. vii-xxvi.